Bargaining In the Presence of a Strategic Ratifier

N. Macartan Humphreys*
Department of Political Science, Columbia University

Abstract

Students of negotiations argue that negotiators benefit from their differences with their constituencies. Attempts to formalize this idea share a feature that is rarely made explicit: they build in an assumption that ratifiers can not, or need not, take account of the effects of their decisions on other actors. Under this assumption, ratifiers affect outcomes if and only if they dislike the *status quo* relative to bargained outcomes. In contrast, this paper considers the case where ratifiers are strategic. In this case, and unlike the non-strategic case, ratifiers affect outcomes even if they prefer all policy options to the *status quo*. Hence we find that past work underestimates the expected effect of ratification. Furthermore, we find that the direction of the influence of ratifiers can be determined under very general conditions: strategic ratifiers generally benefit the most like-minded negotiator and harm the other. For such cases we identify what types of ratifiers are most beneficial for negotiators in particular bargaining games. A unique exception to the beneficial role played by ratifiers is found for cases in which the pie is relatively "indivisible"; in this case a ratification constraint can harm both parties, even when the ratifier prefers all options to the *status quo*.

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*email: macartan.humphreys@columbia.edu. My thanks to Jeffrey Frieden, Bob Bates and Torben Iverson for helpful conversations.

1 Introduction

A conjecture exists that negotiators can benefit from constraints imposed by hawkish constituencies. Such constraints allow a negotiator credibly to claim that some unfavorable proposals that she would find otherwise acceptable are not ratifiable at home. An early statement of the conjecture is given by Thomas Schelling [12, p. 28]:

> [I]f the executive branch negotiates under legislative authority, with its position constrained by law and it is evident that Congress will not be reconvened to change the law within the necessary time period, then the executive branch has a firm position that is visible to its negotiating partners.

Although stated here in terms of the way in which an inanimate constraint—a law—affects a negotiators's calculus, Schelling's conjecture is also relevant to bargaining situations in which, to implement a bargain, a negotiator is constrained by the need to win the support of some third party, who may or may not be strategic. There are many situations in which the argument could be of importance: Schelling motivates the claim initially with reference to negotiations between management and labor, other relevant applications include peace negotiations or trade policy negotiations.

The seemingly wide field of application of the conjecture has led to multiple attempts to assess the generality of its logic and to pin down the mechanism through which the logic works. In one early attempt to provide greater formalism to the conjecture, Putnam [10] made use of the idea of "winsets"—the set of points that the constituency prefers to the *status quo*—to argue that the smaller the winset, the greater the advantage to the negotiator.

Putnam's formulation of the problem has structured most subsequent attempts to analyze the role of ratifiers. Implicit is the idea that smaller winsets rule out bad outcomes. However, since smaller winsets may also rule out good outcomes, it is not clear whether a small winset will in fact benefit or harm a negotiator. These possibly ambiguous effects led to a series of more formal attempts to identify when and how ratifiers matter (see for example [3], [4], [5], [6], [8], [14]). Formal analysis reveals that in contexts where ratifiers are non-strategic, the requirement of ratification may benefit or harm *either* negotiator, depending on the ratifier's evaluation of the *status quo* relative to bargaining outcomes.

These studies inherit a feature from Putnam's formulation of the conjecture that is rarely made explicit but that is now worth emphasizing: Putnam's formulation builds in the idea that ratifiers can not, or need not, take account of the effects of their decisions on other actors—they require parametric but not necessarily strategic rationality. One consequence of this assumption is that ratifiers prove to be relevant in these models if and only if they prefer the *status quo* to outcomes that would otherwise be agreed upon by negotiators.[1] Ratifiers are constrained to take the present—the *status quo*—as the reference point, rather than expectations of possible future deals. A consequence is that we should expect the constraints of negotiaors to be of little importance in situations in which the *status quo* is painful.

How plausible is the assumption that negotiators are strategic but that ratifiers are not (or, at least, that they have no opportunity to employ strategies)? The assumption may be appropriate if ratifiers are faced with a Take-It-or-Leave-It offer. The assumption may also be plausible if the "ratifier" is not in fact capable of strategy. Indeed, as indicated in Schelling's original discussion, the constraint imposed on the negotiator may sometimes derive from inanimate sources such as the legal or moral position of the negotiator rather than from the preferences or powers of human ratifiers.

For a large class of situations, however, the assumption is hard to defend. In many applications, rejection by the ratifier may lead to a re-opening of negotiations rather than simply to an end of negotiations. Indeed, applications in which we may expect to see such behavior are very common: they include international trade negotiations that need domestic ratification, end of civil war peace negotiations in which the agreement of some fighting faction is needed to implement a deal, and bargaining in bicameral legislatures with a presidential veto.

For situations in which ratifiers may behave in this way, present formalizations of the conjecture, that, by their construction, exclude the possibility that ratifiers act strategically, have nothing to say.

To respond to the problem I employ an alternating offers model of bargaining over public goods to determine the impact of strategic ratifiers. From a formal perspective, the model presented is more general than previous work insofar as the outcome space is allowed to be multidimensional and player preferences are not constrained to be linear or Euclidean. From a political perspective, the novelty of the model is that it allows us to consider negotiations in which ratifiers are sufficiently strategic that they may select to forego acceptable offers in the expectation of more favorable future rewards. To allow for better comparison with past work, I restrict attention to situations in which ratifiers find

[1] In [4] the ratifier is partly strategic insofar as it can determine its reservation price endogenously. The process between the bargainers is however solved cooperatively and the ratifiers role is, as in previous work, limited to an up-down vote.

all outcomes preferable to the *status quo*. In doing so, I formally remove the role of winsets and hence remove the mechanism that [10] and subsequent authors focus on to derive Schelling conjecture type results.

The question that the model is designed to answer then is: with the mechanism that has been used to motivate the Schelling conjecture removed, and with ratifiers that have both an agenda and a mind of their own, will the intuitions of the Schelling conjecture find any support? The surprising answer is: *yes*. The conjecture holds in all cases except for a class of games in which negotiators are very impatient and the pie is relatively "indivisible."

2 The Model

Consider the following variation of the Ståhl-Rubinstein infinite-horizons alternating offers framework developed in [13] and [9].[2]

In every odd (alt. even) period $t \in T := \{0, 1, 2, ..., \infty\}$ in which an agreement has not already been reached, Player i (alt. j) proposes an offer $x \in X$, where X is a convex and compact subset of $\mathbb{R}^n$. If x is accepted by j (alt. i), it is put to the ratifier. If x is ratified it is implemented immediately and the game moves on to period $t+1$. The new policy remains in place thereafter and all bargaining ends. If it is rejected by j (alt. i) or fails to receive ratification, then the *status quo* remains in place and the game moves into period $t+1$.

The domain of preference relations is the product $X \times T$ and hence preference relations are described over pairs in which the first element records the element of X and the second the time period in which the element is implemented. With a slight abuse of notation I also treat elements of X alone as the domain of preference relations for situations when a Player is comparing two elements of X for a single time period. With Player 3 acting as ratifier, preference relations $\succsim_i$ satisfy the following conditions:

1. *Disagreement is the worst outcome:* $(x, t) \succ_i SQ$ for all $x \in X$, $t \in T$. In making this assumption I formally remove the role of winsets from the analysis.

2. *Time is valuable:* $(x, t) \succ_i (x, t+1)$ for all $x \in X$, $t \in T$.

3. *Stationarity of Preferences*: $(a, t) \succsim_i (b, t+r)$ if and only if $(a, s) \succsim_i (b, s+r)$ for arbitrary s, t and r.

[2] The assumptions imposed on the bargaining space and on preferences in this paper are closest to those used in [8]. See [1] for a general application of this framework to a policy environment for games in which the proposer is randomly recognized.

4. *Continuity:* The preferences of each player over elements in X may be represented with a continuous von Neumann-Morgenstern utility function.

With $\succsim_i$ representable by a continuous strictly quasi-concave von Neumann-Morgenstern utility function $u_i : X \rightarrow \mathbb{R}^1$, $x \succsim_i y \Leftrightarrow u_i(x) \geq u_i(y)$ and $x \sim_i y$ implies $z \succ_i x$ for every point z in the interior of the convex hull of x and y. As X is compact and convex each player has a most-preferred point, or "ideal point" in X.[3] I use $\breve{x}$ to denote the ideal point for Player 1; and $\breve{y}$ for Player 2. The negotiation set (contract curve) for Players 1 and 2, denoted by $\mathcal{C}$, is defined as $\mathcal{C} := \{x \in X | \nexists y \in X$ such that $y \succsim_i x$ for all i and $y \succ_i x$ for some $i \in \{1, 2\}\}$. From the compactness of X and from (1.) above, we have that $\mathcal{C}$ is non-empty. $\mathcal{C}$ is also one dimensional, connected, compact and strictly monotone: for $x, y \in \mathcal{C}$, $y \succsim_i x$ implies $y \prec_j x$; this in turn implies that there is no pair (x, y) on $\mathcal{C}$ such that $x \sim_i y$ for any i.[4]

I add two further conditions.

Condition $(*)$ For points a, b, c, d on $\mathcal{C}$, with $c \succ_i d$, $(d, 0) \sim_i (b, 1)$ and $(b, 0) \sim_j (d, 1)$:

(I) $(a, 0) \sim_j (c, 1)$ implies $(c, 0) \succ_i (a, 1)$

(II) $(c, 0) \sim_i (a, 1)$ implies $(c, 1) \succ_j (a, 0)$

Informally Condition $(*)$, related to the notion in [9] of "increasing costs to delay," requires that compensating satisfied players for delay is more difficult than compensating dissatisfied players. Hence for example, players who are as dissatisfied with an outcome as they are with the *status quo* are indifferent whether or not they receive that outcome with delay whereas players that prefer an outcome to the *status quo* prefer consuming it immediately to waiting. In Claim 9 in the appendix I show that this assumption holds for example whenever $\mathcal{C}$ is linear and players have concave preferences.

The second additional condition places a weak restriction on relations between the ratifier's preferences and those of the negotiators:

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[3]To confirm that a unique $\breve{x}_i$ exists note that with u_i continuous and X compact, u_i attains a maximum on X. Furthermore this maximum is unique since if two points x^* and y^* were both maxima, any point z in $\text{Int}(\text{Co}(x^*, y^*))$ would lie in X (from the convexity of X) and have $u_i(z) > u_i(x^*)$, from strict quasiconcavity; a contradiction.

[4]To see that $\mathcal{C}$ is strictly monotone, consider two distinct points $x, y \in \mathcal{C}$ such that $y \succsim_i x$ and $y \sim_j x$ (we can ignore $y \succ_j x$ as this is excluded by the definition of $\mathcal{C}$). With strictly quasiconcave preferences there exists a point $z \in \text{Co}(x, y) \subseteq X$ such that $z \succ_j x$ and $z \succsim_i x$. But then $x \notin \mathcal{C}$, a contradiction. For the second claim, note that $y \sim_i x$ implies $y \prec_j x$, which implies $x \prec_i y$, a contradiction.

Condition (∗∗) The ratifier has single peaked preferences over $\mathcal{C}$.

3 Results

I derive results that allow comparison between the outcomes of an alternating bargaining game with and without a ratifier. In Proposition 1 I extend the Ståhl-Rubinstein framework to allow for the possibility of a ratifier and identify constraints on players' strategies that guarantee them to be sub-game perfect equilibrium strategies in the presence of a ratifier. In Proposition 2 I identify conditions where ratifiers are irrelevant to bargained outcomes. In the final two propositions I characterize the effects of ratifiers in cases where they do alter the outcome.

Proposition 1 *There exists a pair* $(\bar{x}, \bar{y})$ *such that:*
$\bar{x}$ *maximizes* $\succsim_1$ *s.t.* (C_1) $(x, 0) \succsim_2 (\bar{y}, 1)$ *and* (C_2) $(x, 0) \succsim_3 (\bar{y}, 1)$
$\bar{y}$ *maximizes* $\succsim_2$ *s.t.* (C_3) $(y, 0) \succsim_1 (\bar{x}, 1)$ *and* (C_4) $(y, 0) \succsim_3 (\bar{x}, 1)$
Furthermore the following constitutes a sub-game perfect equilibrium set of strategies: Player 1 always proposes $\bar{x}$ *and accepts* $\bar{y}$ *and any proposal* y *if and only if* $(y, 0) \succ_1 (\bar{x}, 1)$*; Player 2 always proposes* $\bar{y}$*, accepts* $\bar{x}$ *and any proposal* x *if and only if* $(x, 0) \succ_2 (\bar{y}, 1)$*. The ratifier always ratifies any* x *from Player 1 such that* $(x, 0) \succsim_3 (\bar{y}, 1)$ *and ratifies any* y *offered by Player 2 such that* $(y, 0) \succsim_3 (\bar{x}, 1)$*.*

Proof. Let $f_1(y) : X \to X$ denote the solution to the problem of maximizing $\succsim_1$ s.t. $(x, 0) \succsim_2 (y, 1)$ and $(x, 0) \succsim_3 (y, 1)$. With strictly quasi-concave preferences, the set satisfying $(x, 0) \succsim_2 (y, 1)$ and $(x, 0) \succsim_3 (y, 1)$ is strictly convex and hence $f_1(y)$ is unique. Furthermore, with continuous preference relations, $\succsim_1$ is continuous and the sets satisfying $(x, 0) \succsim_2 (y, 1)$ and $(x, 0) \succsim_3 (y, 1)$ are continuous in y and hence $f_1(y)$ is continuous in y. The analogous function $f_2(x) : X \to X$, where $f_2(x)$ denotes the solution to the problem of maximizing $\succsim_2$ s.t. $(y, 0) \succsim_1 (x, 1)$ and $(y, 0) \succsim_3 (x, 1)$, is also continuous. Hence the function $f(x, y) = (f_1(y), f_2(x)) : X \times X \to X \times X$ is a continuous mapping from a compact convex set into itself, and, from Brouwer's fixed point theorem, has a fixed point. The existence of such a point $(\bar{x}, \bar{y}){=}f(\bar{x}, \bar{y})$ establishes the first part of the claim.

For the remainder of the Proposition, using the one stage deviation principle, we check that a deviation from the prescribed strategies in any single stage does not improve the payoff of any player (see Fudenberg and Tirole 1995, 108-10). For the ratifier, rejecting an offer $\tilde{x}$ from Player 1 in time 0 that she weakly prefers to $(\bar{x}, 0)$ improves the ratifier's payoff only if $(\bar{y}, 1) \succ_3 (\tilde{x}, 0) \succsim_3 (\bar{x}, 0)$, which is not the case. Accepting any offer $\tilde{x}$ in time 0 such that $(\tilde{x}, 0) \prec_3 (\bar{y}, 1)$ is sub-optimal since rejecting guarantees outcome $(\bar{y}, 1)$. An analogous argument applies for offers from Player 2.

For Player 1, offering any $\tilde{x} \neq f_1(\bar{y})$ must either be an offer for which $\tilde{x} \prec_1 \bar{x}$ or else it must be that $\tilde{x}$ is not acceptable to at least one of Player 2 or Player 3. The former is clearly sub-optimal. In the latter case, Player 1 receives $(\bar{y}, 1)$ instead of $(\bar{x}, 0)$. Note however that if $\bar{y} \succ_1 \bar{x}$, then, since $(\bar{y}, 0) \succsim_2 (\bar{y}, 1)$ and $(\bar{y}, 0) \succsim_3 (\bar{y}, 1)$, necessarily, $\bar{x} \neq f(\bar{y})$, a contradiction. It follows then that $(\bar{y}, 1) \prec_1 (\bar{y}, 0) \precsim_1 (\bar{x}, 0)$ and hence that choosing $(\bar{y}, 1)$ over $(\bar{x}, 0)$ is suboptimal. Deviation in any stage where Player 1 has to choose whether to accept or reject an offer made by Player 2 occurs if Player 1 accepts an offer y with $(y, 0) \precsim_1 (\bar{x}, 1)$, or rejects an offer y for which $(y, 0) \succ_1 (\bar{x}, 1)$. In the former case, Player 1 does not improve upon the return he gets from playing rejecting, since rejecting yields $(\bar{x}, 1)$ whereas accepting yields $(y, 0)$ and $(y, 0) \precsim_1 (\bar{x}, 1)$. In the latter case accepting y rather than waiting one period and receiving $\bar{x}$ yields a lower return for sure since $(y, 0) \succ_1 (\bar{x}, 1)$. An analogous argument demonstrates that one stage deviation is also sub-optimal for Player 2. ■

Note that if conditions (C_2) and (C_4) above are removed then the equilibrium pair of strategies $(\bar{x}, \bar{y})$ are those in the standard Ståhl-Rubinstein bargaining game (see for example [9]). Henceforth I use (x^*, y^*) to label these equilibrium offers in the "unconstrained" game and $(\bar{x}, \bar{y})$ to label the equilibrium offers identified above for the constrained game. The last proposition guarantees that such an $(\bar{x}, \bar{y})$ exists and can be used to characterize a sub-game perfect equilibrium set of strategies. An almost identical proof guarantees that indeed *every* pair of offers (x, y) that constitute part of a stationary sub-game perfect equilibrium satisfy the conditions imposed on $(\bar{x}, \bar{y})$ in the statement of the proposition. In the remaining sections, I ask: how do the $(\bar{x}, \bar{y})$ defined in Proposition 2 differ from benchmark outcomes (x^*, y^*).

I begin by describing cases where the ratifier has no effect.

Proposition 2 [***Ratifier Irrelevance***] *Equilibrium offers in the game without a ratifier, (x^*, y^*), are also equilibrium offers in the game with a ratifier if and only if $(x^*, 0) \succsim_3 (y^*, 1) \succsim_3 (x^*, 1)$ or $(y^*, 0) \succsim_3 (x^*, 1) \succsim_3 (y^*, 1)$. In this case the strategies described in the previous proposition with $\bar{x} = x^*$ and $\bar{y} = y^*$ are sub-game perfect equilibrium strategies.*

Proof. See Appendix. ■

The last propositions identifies the conditions that must be satisfied for the ratifier to matter sufficiently to alter the outcome of negotiations.

The final two propositions identify *how* she matters. In these I compare players' attitudes to the pair $(\bar{x}, \bar{y})$ relative to the pair (x^*, y^*). I distinguish between two cases. In the first, the constraints that bargainers place on each other in the unconstrained games bind. In these cases we may expect players to make offers that compromise their ideals. In the second case, the first mover advantage is such that for at least one of the players, when making offers she can afford to offer her ideal.

The next Proposition considers the first case. It finds that when the ratifier prefers Player i's unconstrained offer to Player j's unconstrained offer, then the equilibrium offers in the constrained game are such that Player i is made strictly better off both by her own offer and by Player j's offer relative to offers from the unconstrained game. Player j however is made strictly worse off by both offers, relative to the corresponding offers in the unconstrained game. The Proposition is written for the case where Player 3 prefers Player 2's unconstrained offer. Clearly an analogous result holds however if the ratifier prefers Player 1's offer.

Proposition 3 *If:*
(i) $(x^*, 0) \sim_2 (y^*, 1)$ *and* $(y^*, 0) \sim_1 (x^*, 1)$ *but* $(x^*, 0) \prec_3 (y^*, 1)$
(ii) $y^* \neq \breve{y}$
Then: $\bar{y} \succ_2 y^*$, $\bar{y} \prec_1 y^*$, $\bar{x} \prec_1 x^*$, *and* $\bar{x} \succ_2 x^*$

Proof. To prove the proposition it is sufficient to establish that $\bar{y} \succ_2 y^*$. To see this, observe that if $\bar{y} \succ_2 y^*$, then, using C_1 from Proposition 2 we have $(\bar{x}, 0) \succsim_2 (\bar{y}, 1) \succ_2 (y^*, 1) \sim_2 (x^*, 0)$ and hence $\bar{x} \succ_2 x^*$ and so $\bar{x} \prec_1 x^*$(from strict monotonicity of $\mathcal{C}$). Furthermore $\bar{y} \succ_2 y^*$ implies $\bar{y} \prec_1 y^*$ (from strict monotonicity of $\mathcal{C}$).

Exactly one of C_2 or C_4 from Proposition 2 is binding. This follows since if both are binding then $(\bar{x}, 0) \sim_3 (\bar{y}, 1) \sim_3 (\bar{x}, 2)$ contradicting the assumption that time is valuable. If neither is binding then neither player is constrained in equilibrium and hence $\bar{y} = y^*$ and $\bar{x} = x^*$; but then since $(x^*, 0) \prec_3 (y^*, 1)$, C_2 is not satisfied. With only one of C_2, C_4 binding we can divide all possible equilibriums into three classes: (i) where both C_4 and C_3 are slack (ii) where C_4 is slack but C_3 is binding (iii) where C_4 is binding (and C_2 is slack). Consider each in turn:

(i) With neither C_3 nor C_4 binding, Player 2 is unconstrained in equilibrium and in these instances $\bar{y} = \breve{y} \succ_2 y^*$.

(ii) If C_4 is slack but C_3 is binding then y^* and $\bar{y}$ both lie on $\mathcal{C}$. Assume first that $y^* \sim_2 \bar{y}$. With y^* and $\bar{y}$ on $\mathcal{C}$, $y^* \sim_2 \bar{y}$ implies $y^* = \bar{y}$. However $\bar{x} \neq x^*$ (as in this case C_2 would be violated). If x^* maximizes $\succsim_1$ subject to C_1 (uniquely) then since $\bar{x}$ solves the further constrained problem of maximizing $\succsim_1$ subject to C_1 and C_2 then, with $\bar{x} \neq x^*$, $\bar{x} \prec_1 x^*$. But this implies that $(\bar{y}, 0) \sim_1 (y^*, 0) \sim_1 (x^*, 1) \succ_1 (\bar{x}, 1)$ and hence

$(\bar{y},0) \succ_1 (\bar{x},1)$ contrary to the assumption that C_3 is binding. Assume next that $\bar{y} \prec_2 y^*$ and hence (since both y^* and $\bar{y}$ lie on $\mathcal{C}$) $\bar{y} \succ_1 y^*$. Define $\tilde{x} \in \mathcal{C}$ such that $\tilde{x} \sim_1 \bar{x}$. Note that from strict monotonicity of $\mathcal{C}$ we have $\tilde{x} \succsim_2 \bar{x}$ (where the inequality is strict if $\tilde{x} \neq \bar{x}$). Since $(\bar{y},0) \sim_1 (\bar{x},1)$ we have $(\tilde{x},1) \sim_1 (\bar{y},0)$. But since $\bar{y} \prec_2 y^*$ this implies (from Condition $(*)$) that $(\bar{y},1) \succ_2 (\tilde{x},0) \succsim_2 (\bar{x},0)$ and hence that $\bar{x}$ does not satisfy C_1, a contradiction. Hence $\bar{y} \succ_2 y^*$.

(iii) The remaining cases are those in which C_2 is slack and C_4 is binding, and hence $\bar{x}$ lies on $\mathcal{C}$. Assume that $\bar{y} \precsim_2 y^*$.

Observe that $\bar{x} \succsim_1 x^*$. To see this note that if C_1 is binding then $(\bar{x},0) \sim_2 (\bar{y},1) \precsim_2 (y^*,1) \sim_2 (x^*,0)$ implies that $\bar{x} \precsim_2 x^*$and hence (since $\bar{x}$ lies on $\mathcal{C}$), $\bar{x} \succsim_1 x^*$. If instead C_1 is slack then Player 1 is unconstrained in equilibrium and chooses her ideal, $\bar{x} = \breve{x} \succsim_1 x^*$.

Furthermore, $\bar{x} \succsim_1 x^*$ implies $\bar{x} \prec_3 y^*$. To see this note that with $\bar{x} \succsim_1 x^*$, we have that $\bar{x}$, x^* and y^* are ordered on $\mathcal{C}$ with $x^* \in [\bar{x}, y^*)$. From Condition $(**)$, $(\bar{x},0) \succsim_3 (y^*,1)$ implies $(x,0) \succsim_3 (y^*,1)$ for all $x \in [\bar{x}, y^*)$. But since x^* lies in this range, we have $(x^*,0) \succsim_3 (y^*,1)$, contradicting $(x^*,0) \prec_3 (y^*,1)$. Hence $(\bar{x},0) \prec_3 (y^*,1)$ and so $\bar{x} \prec_3 y^*$.

Now define the point $\tilde{y}$ that maximizes $\succsim_2$ subject to $(y,0) \succsim_1 (\bar{x},1)$. Note that since $(\bar{x},1) \succsim_1 (x^*,1) \sim_1 (y^*,0)$, we have $(\bar{x},1) \succsim_1 (y^*,0)$ and hence that no point y such that $y \prec_1 y^*$ satisfies C_3. Offer $\tilde{y}$ then must lie on $\mathcal{C}$ between $\bar{x}$ and y^*. But from quasi-concavity, the ratifier strictly prefers all points in $(\bar{x}, y^*)$ to $\bar{x}$, including $\tilde{y}$. Hence $\tilde{y}$ satisfies C_4 with slack. Since $\tilde{y}$ maximizes $\succsim_2$ subject to C_3 and it also satisfies C_4, then it maximizes $\succsim_2$ subject to C_3 and C_4. Hence $\tilde{y} = \bar{y}$ and C_4 is slack, contrary to our assumption in Case (iii). ■

The Proposition highlights two features. The first is the role of interests. The ratifier alters the outcome because she strictly prefers what one negotiator offers in the unconstrained game to the offer of the other negotiator. If we assume that the ratifier is more likely to accept the bargained outcome of her own team than that proposed by the other team, then the proposition provides strong support for the Schelling conjecture. Conversely, if the interests of the ratifier were orthogonal to those of the negotiators then the power of ratification is irrelevant. The second is the role of time: more patient ratifiers have a greater impact.

The final proposition considers the second case in which time is highly valuable to negotiators but the divisibility of the pie is low.[5] In these games a proposer can offer her own ideal and have this accepted in

[5]Informally, the divisibility of a pie in this context refers to the range of possible efficient partitions of the value added of a deal. Hence for example if preferences were such that the only efficient divisions of a dollar awarded between $.40 and $.60 to Player 1, this pie is less divisible than that in a game in which any offer to Player 1

equilibrium. As before, the presence of the ratifier worsens the payoffs for the negotiator from the other side. However, in this instance a patient ratifier whose interests differ sufficiently from those of her own negotiator may also worsen the prospects for the negotiator from her own side.

Proposition 4 *Assume* $(x^*, 0) \succsim_2 (y^*, 1)$ *and* $(y^*, 0) \succsim_1 (x^*, 1)$*;* $y^* = \breve{y}$ *and* $x^* = \breve{x}$*. Then:*
(i) If $(x^*, 0) \prec_3 (y^*, 1)$ *then* $\bar{y} = y^*$ *and* $\bar{x} \prec_1 x^*$*.*
(ii) If $(y^*, 0) \prec_3 (x^*, 1)$ *then* $\bar{x} = x^*$ *and* $\bar{y} \prec_2 y^*$*.*

Proof. Consider (i). Note that since x^* uniquely maximizes $\succsim_1$ and $\bar{x} \neq x^*$ (as, otherwise C_2 would not be satisfied) we have $\bar{x} \prec_1 x^*$. Since y^* maximizes $\succsim_2$, in order to check that $\bar{y} = y^*$ we need simply to check that $(y^*, 0) \succsim_1 (\bar{x}, 1)$ and $(y^*, 0) \succsim_3 (\bar{x}, 1)$. Indeed, the first of these constraints is satisfied with slack, as $(y^*, 0) \succsim_1 (x^*, 1) \succ_1 (\bar{x}, 1)$ implies that $(y^*, 0) \succ_1 (\bar{x}, 1)$. The constraint imposed by Player 3 is also satisfied with slack. To establish this, assume to the contrary that $(\bar{x}, 1) \succsim_3 (y^*, 0)$. In this case $(\bar{x}, 0) \succ_3 (\bar{x}, 1) \succsim_3 (y^*, 0) \succ_3 (y^*, 1)$ implies $(\bar{x}, 0) \succ_3 (y^*, 1)$. However, $(\bar{x}, 0) \succ_3 (y^*, 1)$ implies that some point, $\tilde{x}$, on the line interval $(\bar{x}, x^*]$, satisfies $(\tilde{x}, 0) \succsim_3 (y^*, 1)$. From quasi-concavity, $(x^*, 0) \succsim_2 (y^*, 1)$ and $(\bar{x}, 0) \succsim_2 (y^*, 1)$ imply $(\tilde{x}, 0) \succsim_2 (y^*, 1)$. Also from strict quasi-concavity and the fact that $x^* \succ_1 \bar{x}$, $\tilde{x} \succ_1 \bar{x}$. Hence $\bar{x}$ can not be the maximizer of $\succsim_1$ subject to $(x, 0) \succsim_2 (y^*, 1)$ and $(x, 0) \succsim_3 (y^*, 1)$, a contradiction. We have then that $(\bar{x}, 1) \prec_3 (y^*, 0)$.

The proof of (ii) is identical subject to a re-labelling of players. ■

Proposition 4 guarantees that $x^* \succ_1 \bar{x}$ but not that $\bar{x} \succ_2 x^*$. Hence in situations where the ratifier prefers Player 2's unconstrained offer with delay to Player 1's offer without delay, Player 2 does as well as before in games in which she offers first. In games where Player 1 offers first Player 1 certainly loses out. Possibly however Player 2 may also lose out when Player 1 offers first if Player 1's attempts to placate the ratifier lead her to propose a point $\bar{x}$ with $(x^*, 0) \succ_2 (\bar{x}, 0) \succsim_2 (y^*, 1)$.

In such games then the presence of a ratifier may make matters worse for *both* players. The fact that Player 2 can make Player 1 an offer that is worse for both than the offer in the unconstrained game is due to the fact that, in these games, both players have complementary interests insofar as each *strictly* prefers the other's offer to waiting a period for her own offer. In other words, the constraints that they impose on each

in the range \$0 to \$1 may be efficient. For spatial games a pie is partially indivisible if for one or both players the *status quo* is distant relative to the ideal of the other player.

other are slack— that they do not fare worse in the unconstrained game is because there is sufficient commonality of interests for each not to desire to push the other to the boundaries of their acceptance set. This slack however may be consumed when Player 2 is required to make an offer that is acceptable to the ratifier.

4 Examples

I end this paper by considering three examples that illustrate some of these results on bargaining in the presence of ratifiers. The first example, of a Divide-the-Dollar game, illustrates the roles of preference dependence and patience. The second provides a case where the pie is partly indivisible and shows how both negotiators may lose out due to the presence of a ratifier. While Propositions 3 and 4 determine when a ratifier matters and who benefits as a result, without further constraining utility functions we are unable to say *how much* different players benefit from the presence of a ratifier. The final example provides an answer to this question for a common class of games. In all three cases I assume that each player's preferences over $X \times T$ may be represented with a utility function over X and time-constant discount rate, δ_i.

Example 5 ***Benefits to a Negotiator in Divide-the-Dollar Games.*** *Two players negotiate over a three-way split of a dollar subject to ratification. The ratifier receives some share, ε, of Player 2's share.*

In a Divide-the-Dollar game, if gains are purely private ($\varepsilon = 0$), then a ratifier has no effect on the bargained outcome and achieves nothing. In this instance the Rubinstein solution applies and, assuming linear utility and discount rate, δ_i for each negotiator, Player 1 offers $\$\frac{\delta_2 - \delta_1\delta_2}{1-\delta_1\delta_2}$ to Player 2; Player 2 offers $\$\frac{\delta_1 - \delta_1\delta_2}{1-\delta_1\delta_2}$; each offers nothing to the ratifier and each accepts the other's offer (or better). If instead the gains are at least partly public ($\varepsilon > 0$, but arbitrarily small), then a patient ratifier improves the effectiveness of a negotiator. In this case ratification will be relevant if and only if $\delta_R > \delta_2$. If $\delta_R > \delta_2$ then Player 2 offers $\$\frac{\delta_1 - \delta_1\delta_R}{1-\delta_1\delta_R}$ while Player 1 offers $\$\frac{\delta_R - \delta_1\delta_R}{1-\delta_1\delta_R}$. The outcome is exactly as if Player 1 were negotiating directly with the ratifier, ignoring Player 2. Player 2 does strictly better relative to the negotiations without a ratifier whenever the ratifier is more patient than she is. A negotiator then would do well to promise a share of his takings to a patient ratifier.

Example 6 ***Both Negotiators Are Made Worse off by the Ratifier.*** *Consider the game played over $[0,1]^2$ in which Player 1's Utility is* *given by* $u_1(x) = 10-(1-x_1)^2-5(x_2)^2$*, Player 2's utility is* $u_2(x) = 10-5(x_1)^2-(x_2)^2$ *and the Ratifier's utility is* $u_R(x) = 10-(1-x_1)^2-(1-x_2)^2$*. Set discount rates* $\delta_1 = \delta_2 = .4$ *and* $\delta_R = .99$*.*

In this example preferences are such that the relative salience of Dimension 2 to Dimension 1 is greatest for Player 1 and lowest for Player 2. In the unconstrained game, Players 1 and 2 offer their ideals, $(1,0)$ and $(0,0)$ respectively and accept any offer x such that $u_i(x) \geq 4$, for $i \in \{1,2\}$. When Player 2 offers the negotiators receive $u_1 = 9$ and $u_2 = 10$. In the constrained game, since $\delta_R u_R(1,0) = 8.91 > u_R(0,0) = 8$, the ratifier would rather wait to receive an offer of $(1,0)$ from Player 1 than to accept an offer of $(0,0)$ from Player 2. In this case, while Player 1 may play the same strategies as before, Player 2 must alter her strategy in order to make an offer acceptable to the ratifier, at least cost to herself. This is done by offering $(.13,.42)$. This offer just satisfies the ratifier. And it does so by yielding on dimension 2, which, while optimal for Player 2, is especially costly for Player 1. When Player 2 makes this offer, the negotiators receive $u_1 = 8.3$ and $u_2 = 9.7$, a worsening for both players relative to the unconstrained game.

Example 7 ***Euclidean Spatial Games****.* *Consider the game where* $X = [-2,3]\times[-2,2] \subset \mathbb{R}^2$*, players* i *and* j *have ideals* $\breve{x}_i = (1,0)$ *and* $\breve{x}_j = (0,0)$ *and all players have utility* $u_k(x) = 10-|x-\breve{x}_k|^2$ *and common discount rate* $\delta = .95$*. Let* $\breve{x}_3$ *vary over the range of* X*.*

The choice of ideals in this example is arbitrary but in this instance allows us to distinguish easily between the dimension along which negotiators disagree (the first dimension) and the dimension along which negotiators agree (the second).

Figure 1 reports the payoffs from this game to Players i and j from i's equilibrium offer given the presence of a ratifier with ideal $\breve{x}_3 \in X$. The results show that "hawkish" ratifiers are better than dovish ratifiers—where by hawkish we mean that ratifiers have extreme preferences *on the dimension along which negotiators disagree*. Indeed as long as all points in X are preferred by the ratifier to the *status quo*, the more hawkish the better—in these instances the negotiator benefits from disagreement within her own camp. Conversely, a negotiator benefits from homogeneity within the opposing camp. The effects of discord along the dimension along which negotiators agree are just the opposite: Negotiators benefit more from ratifiers that agree with them along this dimension. If there is discord within a camp along the dimension of agreement, then the opposing camp can satisfy ratifiers by making concessions to the ratifier

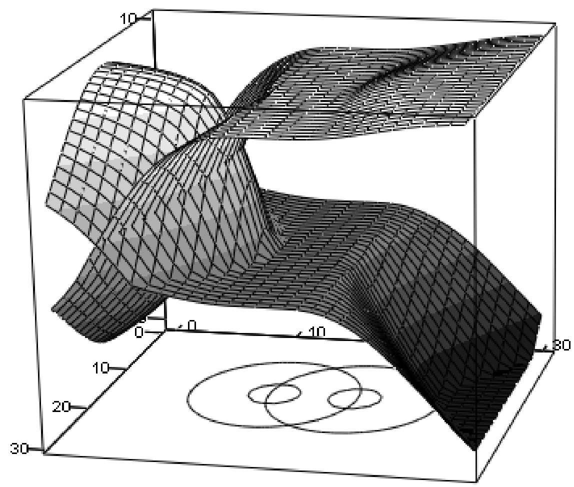

Figure 1: This figure shows payoffs to i and j from i's equilibrium offer as a function of the location of the ideal of the ratifier on X. The floor of the graph represents X and has sample indifference curves marked for i and j, with i's ideal to the right of j's. The graph that is increasing from left to right is i's payoff for every position of the ratifier's ideal. The graph decreasing from left to right is j's payoff.

that do not benefit the negotiator. This counterintuitive result implies that a negotiator can benefit in particular from her own disagreement with the other camp's ratifier.

It is worth remarking that in calculating equilibrium offers, we may observe which of Constraints $C_1 - C_4$ are binding for different values of $\breve{x}_3$. We find that equilibria exist in which each negotiator is constrained by the other and one of them is also constrained by the ratifier; in which one negotiator is constrained by the ratifier but not by the other negotiator; and in which one or other negotiator is entirely unconstrained in equilibrium. For empirical work, such variation identifies which players "matter" for other players during the negotiations.

5 Conclusions

In this paper I consider games where bilateral bargainers are constrained by a strategic ratifier. The model studied is appropriate for contexts where failure to reach agreements is costly for all parties—including the ratifier—but where the content of agreements is sufficiently important for ratifiers that they try to use their power of ratification to alter the behavior of negotiators. Surprisingly, when the mechanism that has traditionally been used to explain the effect of ratifiers—winsets—is removed, and in cases where the ratifier differs from the negotiator both in terms of her desires and in terms of her strategy, we find very strong support for the notion that ratification constraints help negotiators.

However, while the results are consistent with common intuitions about ratifier effects, the politics underpinning the results is quite different from those produced in past work. When the ratifier alters outcomes, she imposes a constraint on the opposition negotiator that is binding in equilibrium. In some instances the constraint may be so severe that the constraint imposed by her own negotiating team on the opposition negotiators is slack. In these cases it is as if the opposition were negotiating directly with the ratifier: the preferences of the home negotiator—perfectly satiated at the ensuing bargaining point—are irrelevant on the margin. A second result relates to the role of homogeneity and heterogeneity within a single group. Negotiators may benefit from internal dissension within their own group when that dissension produces ratifiers that are extreme, relative to the negotiator, on the dimension along which negotiators disagree with each other; however they benefit from group homogeneity on the dimension along which negotiators agree—internal dissension on this dimension may allow the rival team to placate ratifiers without providing benefits to the negotiator. A third result is that the key determinant of the impact of a ratifier is, in this context, not simply her preferences, but her patience. In studying ratification processes, then, the winsets that are traditionally stressed may not be the most important mechanism underlying the Schelling conjecture; rather, attention should be turned to the implications of the different ways that players value time.

6 Appendices

Claim 9 demonstrates that Condition ($*$) holds for any concave utility functions when contract curves are linear. The proof makes use of the following Lemma:

Lemma 8 *Let w, x, y, z denote four distinct points on $\mathbb{R}^1$ with $w < x < z$ and $w < y < z$ and let $f : \mathbb{R}^1 \to \mathbb{R}^1$ denote a concave function. Then:* $\frac{f(x)-f(w)}{x-w} \geq \frac{f(z)-f(y)}{z-y}$.

Proof. Without loss of generality assume that $w = f(w) = 0$. The chord joining w and z has slope $\frac{f(z)-f(w)}{z-w} = \frac{f(z)}{z}$ and intercept $f(w) = 0$; values on the chord corresponding to any point $w \in \mathbb{R}^1$ are described by the function $g : \mathbb{R}^1 \to \mathbb{R}^1$ where $g(w) = \frac{f(z)}{z}w$. Note that with x and y both lying between w and z we have from the concavity of f that $f(x) \geq g(x)$ and $f(y) \geq g(y)$. Now, $f(y) \geq g(y) = \frac{f(z)}{z}y$ implies that $zf(y) \geq yf(z)$ and hence, subtracting $z(f(z))$ from both sides, $z(f(y) - f(z)) \geq f(z)(y - z)$ which, with $z > y$, implies that $\frac{f(z)}{z} \geq \frac{f(z)-f(y)}{z-y}$. Also, $f(x) \geq g(x) = \frac{f(z)}{z}x$ implies that $\frac{f(x)}{x} \geq \frac{f(z)}{z}$. Together these imply that $\frac{f(x)-f(w)}{x-w} = \frac{f(x)}{x} \geq \frac{f(z)-f(y)}{z-y}$. ■

Claim 9 *Let $\mathcal{C}$ be representable as a compact convex subset of $\mathbb{R}^1$ and let preferences over elements of $\mathcal{C}$ be represented with a concave utility function for each of i and j with $u_i' > 0$, $u_j' < 0$ and $u_k(x) > 0$ for $x \in \mathcal{C}$, $k \in \{i, j\}$. Then, for points a, b, c, d on $\mathcal{C}$, with $c > d$, $\delta \in (0, 1)$, $u_i(d) = \delta u_i(b)$ and $u_j(b) = \delta u_j(d)$:*
(I) $u_j(a) = \delta u_j(c)$ implies $\delta u_i(a) < u_i(c)$
(II) $u_i(c) = \delta u_i(a)$ implies $u_j(a) < \delta u_j(c)$.

Proof. Let the negotiation set and utilities be as in the statement of the claim and note that $c > d$ implies $u_j(d) > u_j(c)$ and $u_i(d) < u_i(c)$.

Assume first that $u_j(a) = \delta u_j(c)$. In this case $u_j(b) = \delta u_j(d)$ implies $b > d$ and $u_j(a) = \delta u_j(c)$ implies $a > c$. Furthermore, $c > d$ implies $a > b$ (since $c > d \Rightarrow u_j(d) > u_j(c) \Rightarrow u_j(b) = \delta u_j(d) > \delta u_j(c) = u_j(a) \Rightarrow a > b$). The slope of the chord between $(d, u_j(d))$ and $(b, u_j(b))$ is $\frac{u_j(d)(\delta-1)}{b-d}$; the slope between $(c, u_j(c))$ and $(a, u_j(a))$ is $\frac{u_j(c)(\delta-1)}{a-c}$. With u_j concave, $d < c$ and $b < a$ we have from Lemma 8 that $\frac{u_j(d)(\delta-1)}{b-d} \geq \frac{u_j(c)(\delta-1)}{a-c}$. Hence, with $\delta < 1$, $\frac{u_j(d)}{b-d} \leq \frac{u_j(c)}{a-c}$. But, then $u_j(d) > u_j(c)$ implies $a - c < b - d$. The slope of the chord between $(d, u_i(d))$ and $(b, u_i(b))$ is $\frac{u_i(d)\frac{1-\delta}{\delta}}{b-d}$; the slope between $(c, u_i(c))$ and $(a, u_i(a))$ is $\frac{u_i(a)-u_i(c)}{a-c}$. If, contrary to the proposition, $\delta u_i(a) \geq u_i(c)$, then $u_i(a) - u_i(c) \geq \frac{1}{\delta}u_i(c) - u_i(c) =$

$u_i(c)\frac{(1-\delta)}{\delta}$, and again using the concavity of u_i we then have that $d < c$ and $b < a$ imply $\frac{u_i(d)\frac{1-\delta}{\delta}}{b-d} \geq \frac{u_i(a)-u_i(c)}{a-c} = \frac{u_i(c)\frac{1-\delta}{\delta}}{a-c}$ and so $\frac{u_i(d)}{b-d} \geq \frac{u_i(c)}{a-c}$. As $u_i(d) < u_i(c)$ we have then that $b-d < a-c$, a contradiction that proves Part I of the claim.

The proof for Part II is essentially identical: $\delta u_i(a) = u_i(c)$ implies $a > c$ and $c > d$ implies $b < a$ (since $c > d \Rightarrow u_i(a) = \frac{1}{\delta}u_i(c) > \frac{1}{\delta}u_i(d) = u_i(b) \Rightarrow a > b$). With u_i concave, $d < c$ and $b < a$ we have $\frac{u_i(d)\frac{(1-\delta)}{\delta}}{b-d} \geq \frac{u_i(c)\frac{(1-\delta)}{\delta}}{a-c}$ and hence $\frac{u_i(d)}{b-d} \geq \frac{u_i(c)}{a-c}$ and so, using, $u_i(d) < u_i(c)$ we have $a - c > b - d$. If, contrary to the proposition $u_j(a) \geq \delta u_j(c)$, then $u_j(a) - u_j(c) \geq u_j(c)(\delta - 1)$ and, again using the concavity of u_j we then have $\frac{u_j(d)(\delta-1)}{b-d} \geq \frac{u_j(a)-u_j(c)}{a-c} \geq \frac{u_j(c)(\delta-1)}{a-c}$, and hence $u_j(d) > u_j(c)$ implies $a - c < b - d$—a contradiction, proving Part II of the claim. ■

Restatement of Proposition 2 [**Ratifier Irrelevance**] *Equilibrium offers in the game without a ratifier, (x^*, y^*), are also equilibrium offers in the game with a ratifier if and only if* $(x^*, 0) \succsim_3 (y^*, 1) \succsim_3 (x^*, 1)$ or $(y^*, 0) \succsim_3 (x^*, 1) \succsim_3 (y^*, 1)$. *In this case the strategies described in the previous proposition with $\bar{x} = x^*$ and $\bar{y} = y^*$ are sub-game perfect equilibrium strategies.*

Proof. Assume without loss of generality that $(y^*, 1) \succsim_3 (x^*, 1)$. To check the *if* part, note that $(x^*, 0) \succsim_3 (y^*, 1) \succsim_3 (x^*, 1)$ implies that rejecting an offer of x^* from Player 1 and then accepting y^* from Player 2 one period later does not improve the ratifier's payoff. With $(y^*, 1) \succsim_3 (x^*, 1)$, $(y^*, 0) \succsim_3 (x^*, 1) \succsim_3 (y^*, 1)$ implies that $(x^*, 0) \sim_3 (y^*, 0) \succ_3 (x^*, 1) \sim_3 (y^*, 1)$ and hence $(x^*, 0) \succ_3 (y^*, 1)$ and so again forgoing x^* to accept y^* one period later does not improve the ratifier's payoff. Similarly $(y^*, 1) \succsim_3 (x^*, 1)$ implies $(x^*, 1) \prec_3 (y^*, 0)$ and hence deviating for one stage after an offer from Player 2 is sub-optimal. With the ratifier accepting their offers from the unconstrained game, the incentives faced by negotiators are unaltered from the unconstrained game. Since no player has an incentive to deviate from their strategies in any single period, the one stage deviation principle implies that these strategies are indeed sub-game perfect. For the *only if* part, note that if $(y^*, 1) \succ_3 (x^*, 0)$ then rejection of x^* and subsequent acceptance of y^* is preferable to accepting x^* immediately. Hence accepting Player 1's offer is not optimal for the ratifier. But in this case, the offers x^* and y^* are not optimal offers for Players 1 and 2 either. To check this assume that they are. In this case Player 2's acceptance set, $\mathcal{A}_2^*$, must include all points x such that $(x, 0) \succsim_2 (y^*, 1)$. In any sub-game perfect equilibrium in which Player 2 offers y^*, Player 3's acceptance set for offers from Player 1 is the set $\mathcal{A}_3^*=\{x|\ (x, 0) \succsim_2 (y^*, 1)\}$. Hence, Player 1 will deviate from her strategy if there exists any point x' such that $x' \in \mathcal{A}_3^* \cap \mathcal{A}_2^*$ and $(x', 0) \succ_1 (y^*, 1)$. But $x' = y^*$ is one such point. Hence if the condition fails the offers in the equilibrium of the constrained game of the Players cannot be (x^*, y^*). ■

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