Policy Dynamics and Legislative Bargaining

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March 18, 2007

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Abstract

This paper presents a dynamic model of legislative bargaining with an evolving default policy. We show that even with a fixed proposer proposal power is limited in equilibrium. Moreover, legislators have induced preferences over the distribution of benefits. We then apply the model to entitlement policies and models of public good production. Our results provide an explanation for the "ratchet effect" of public spending in multi-party parliamentary democracies.

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1 Introduction

Recent theoretical and empirical studies on comparative constitutions have deepened our understanding of how political institutions shape economic policies. For example, models by Persson and Tabellini (1999), Lizzeri and Persico (2001), and Milesi-Ferretti et all (2002) compare how different electoral rules lead to different fiscal policies like the size of general public goods, targeted transfers, local public goods, and corruption. Pagano and Volpin (2006) investigate how electoral rules shape policies on corporate governance. Economic effects of other dimensions of political institutions have also been studied. Persson and Tabellini (2000) compare how different forms of government, presidential regime and parliamentary regime, lead to different fiscal policy outcome. More recently, Battagilini and Coate (2005, 2006) investigate the relationship between legislative bargaining, public investment and debt; and Fong (2006) and Baron, Diermeier, and Fong (2007) show how coalition formation and voting under proportional representation can lead to policy inefficiency. In the empirical literature, some of the theoretical predictions have been seriously tested and new relationships established. For example, Persson and Tabellini (2001, 2003, 2004) create a comprehensive data set on political institutions and empirically investigate how different constitutional arrangements shape fiscal policies.

Most of the studies, however, are based on static models or focus on static policy issues like the sizes of total government expenditure, welfare expenditure, and corruption. This is in marked contrast to the earlier generations of political economy with their emphasis on political business cycles. However, these earlier models relied on very simplified models of political decision-making, such as the median voter theorem, and therefore could not capture constitutional differences. While Persson and Tabellini (2001, 2003, 2004) provide some preliminary empirical studies of the constitutional effects on political business cycles, fiscal deficits and responsiveness of government and welfare expenditures to business cycles, there are very few theories that account for the relationship between constitutions and policy dynamics.[1] The main difficulty is the lack of political decision-making models, particularly ones that are rich, yet flexible enough to capture dynamic environments, i.e., models with changing economic state variables. The application of standard legislative bargaining approaches in multiperiod models usually generate discontinuity and non-concavity of value

[1] Some recent exceptions include Battaglini and Coate (2006, 2007).

functions and policy rules that make the characterization of an equilibrium a challenging task (Baron and Herron 2003, Baron, Diermeier and Fong 2007).

In this paper, we propose a new analytical framework of legislative bargaining. It is characterized by two key features: (1) A policy, once enacted, is in effect until a new law is made. (2) Any legislator with agenda-setting power is allowed to make a new policy proposal at any time and as frequent as possible. The second feature distinguishes our model from all others in the literature.[2] While most dynamic legislative bargaining models are extremely difficult to solve, our model is tractable and exhibits continuous value functions, a rarity in models of collective choice. The model is extremely tractable and at the same time it yields predictions that seem to be consistent with daily observations and recent empirical studies on the value of proposal power (Knight 2005).

In this paper we focus on a particular structure of agenda-setting that is typical of parliamentary democracies. Comparative scholars have long observed that compared to presidential systems the constitutional features of parliamentary systems lead to high levels of agenda control for the executive, i.e. the cabinet (Doering 1995). In many cases, that power is concentrated within the prime minister. We capture this feature formally by considering a single, persistent agenda-setter during a given legislative period. Surprisingly, this framework does not necessarly lead to extreme but rather constrained proposal power.

Specifically, we show that in the context of distributive policy, legislators have indirect preferences over distributions of benefits. What other legislators receive matters to any given legislator, not because of altruistic preferences, but because current distributions affect bargaining power in the future. As a consequence, in equilibrium, the legislators not included in the winning coalition are not fully expropriated, and the value of agenda-setting can be smaller than what is predicted in other proposer models such as Baron and Ferejohn (1989) or Bernheim et al. (2006). This result of constrained proposal power is consistent with some recent empirical findings, for example, Knight (2005).

We then apply our modeling modeling framework to a fully dynamic setting to capture richer political economy environments including the source of policy inertia, especially in the context of entitlement programs and the ratchet effect of government spending in parliamentary democracies with proportional representation (Persson and Tabellini (2004).

Our key bargaining institution is rooted in the model of Baron and Ferejohn (1989).

[2] The relationship with a recent model by Bernheim et al. (2006) is discussed below.

They analyze how legislators bargain over a pie with majority rule and find a unique symmetric stationary equilibrium where only a bare majority of legislators receive positive shares of the pie. The seminal paper is recently tested by Knight (2005) using US data on the distribution of transportation projects. The evidence supports the key qualitative prediction that proposal power is valuable, however, support for the qualitative prediction of proposal power is more mixed. In our model, we show that the possibility to reconsider a policy issue substantially weakens the proposal power for an agenda setter who is conferred power to make policy proposals throughout the whole legislative session. Fearing that the agenda setter uses his power to expropriate in the future, the other legislators do not approve any policy that substantially lower the reservation values of some others.

The paper belongs to the literature of dynamic legislative bargaining with a moving status quo where intertemporal tradeoff between current legislative and future status quo may lead to various patterns of policy dynamics in different contexts. With one-dimensional policy space and single-peaked preferences, Baron (1996) shows that, in the long run, the policy will converge to the alternative preferred by the median voter. Baron and Herron (2003) and Fong (2004) study the game in a multidimensional ideological space. In models of a parliamentary democracy with proportional representation, Fong (2006) shows that an incumbent coalition government strategically manipulates to lower the bargaining position of the outside parties in order to create cheap coalition partners in the future. The incentive leads to more non-central policy outcomes and political failure. Baron et al (2007) show that, with strategic voters, the problem of inefficiency is worsened, since a more extreme status quo that favors the incumbent parties helps it to obtain electoral advantage in a model of strategic voting.

Kalandrakis (2004) analyzes an infinitely repeated divide-the-dollar bargaining game with the same bargaining institution, assuming three players with linear utility. The Markov perfect equilibrium in his model is such that irrespective of the discount factor or the initial division of the dollar, the proposer eventually extracts the whole dollar in all periods. In contrast, in the dynamic version of our model, full expropriation by the agenda setter rarely occurs. The distribution is more egalitarian.

Bernheim et al (2006) examines legislative policy making in institutions with real-time agenda setting and evolving default. Assuming finite rounds of proposal-making and voting within a pork barrel model of redistributive politics, the last proposer is able to pass his favorite policy under relatively weak conditions. As a consequence, the final policy outcome is highly unequal, and the last proposer is able to obtain his ideal policy. As the authors point out in the concluding section, it is natural to wonder whether particular procedures effectively promote a more egalitarian distribution of political power. Our model keeps the setups of evolving default and real-time legislation, but assumes an agenda setter with persistent power throughout the legislative session and no well-defined last round of negotiation. The new elements work together and substantially limit the extent to which the agenda setter can expropriate the other legislators.

This paper is also linked to a recently emerging literature on the role of lack of commitment in policy making. While it is commonly accepted that lack of commitment by the policy maker is a source of inefficiency, our model shows that lack of commitment by the agenda-setter who holds power for a certain amount of time in fact leads to more egalitarian division of social surplus. If we were to introduce some concavity into the legislators' payoff functions, this would imply more efficient policy outcomes.

2 The Core Model

2.1 The Setup

The political system is characterized by a legislature with three members, indexed by $l \in \{1, 2, 3\}$. The legislature collectively decides on how to divide a total benefit of $G \in \mathbb{N}$ units. A feasible policy is therefore a triple $g = (g_1, g_2, g_3)$ with $g_l \in \mathbb{Z}_+$ for all $l \in \{1, 2, 3\}$ and $\sum_{l=1}^{3} g_l \leq G$. Denote the policy space by $\Delta(G)$. The assumption of a discrete policy is made for technical convenience. The units can be as small as necessary, e.g. one cent. In any realistic distributive policy project, e.g. transportation or public works bills, this assumption is satisfied (Knight 2005). The same holds true if the policy space is interpreted as the distribution of office-holding benefits among ministers and their budget. To facilitate discussion of some numerical examples, we assume that G is a multiple of 6, so that the total benefit could be equally divided by any two legislators or all three legislators. None of our main results relies on this assumption.

Every legislator derives utility from the benefit he receives according to the policy. The utility function, assumed to be linear, is given by $u_l(g) = g_l$ for each legislator l.

Legislative decision-making proceeds as follows. One legislator is exogenously assigned to be the sole agenda setter a; i.e. the legislator to make a policy proposal from the policy space. The agenda setter can make proposals any time and as frequently as possible before the session ends. Initially there is a default policy $g^0 \in \Delta(G)$. A default is the policy that will be implemented if no new policy proposal is passed subsequently. There are potentially infinite countable rounds of policy making. In each round, the agenda setter can choose to make a new policy proposal or to pass. Once a proposal is made, it is voted on against the default policy in that round. Voting is by majority rule. If a new policy passes it becomes the new default policy for the next round, otherwise the original default remains.

There are two ways to end the legislative session. First, at the end of each round of negotiation, the session *exogenously* ends with probability $(1 - \delta)$. In other words, the probability of continuing the session and revisiting the policy issue is δ. Second, the session ends *endogenously* if the default policy is such that the agenda setter no longer wants to propose a new policy to defeat it. Once the legislative session ends, the current default policy is implemented as the final policy outcome.

We make two behavioral assumptions. First, the agenda setter makes a policy proposal if and only if doing so makes him strictly better off; otherwise he passes. This assumption can be justified by an infinitesimal cost of agenda setting. Second, a legislator votes against a policy proposal if and only if passage of the bill makes him strictly worse off.

The assumption of an evolving default is similar to the approach proposed by Bernheim et al (2006). That is, the passage of a bill does not prevent the legislature from revisiting the issue at a later date; rather, it changes the default for subsequent deliberations. Bernheim et al. assume an exogenously fixed, commonly known number of bargaining rounds. In our model there is not a well-defined last round. Rather, the number of actual bargaining rounds is endogenously determined in equilibrium.

2.2 Equilibrium Definition

Throughout the analysis we assume stationarity. That is, we restrict analysis to cases in which the agenda setter conditions his proposal strategy only on the prevailing default policy. Consider an arbitrary round of bargaining with default $g \in \Delta(G)$. Let $U_l(g, a)$ be legislator l's reservation value.[3] This is the expected utility of legislator l if no new policy

[3] In the base-line model the agenda setter is exogenous and fixed over time, so, strictly speaking, is not necessary to formulate the value functions and policy rule as functions of a. However, we include "a" among

is made in this round of negotiation.

We do not explicitly model voting strategies. Instead, given our behavioral assumptions on proposal making and voting, any new policy the agenda setter proposes always satisfies at least one of the other legislators by his reservation value. We model voting by an incentive compatibility constraint and only have to specify the proposal strategy of the agenda setter. [[Pohan, this needs more detail, what does this mean? Why do we make this assumption? How limiting is it? => **Pohan's Response:** I do not have time to add more words in this paragraph. Will do it after the conference. This is about how we formulate the model into a mathematical problem, but nothing about assumptions. This requires a new proposal to make at least one other legislator weakly better off. Otherwise the proposal would be rejected by majority voting, and proposing a policy that would be rejected for sure is nothing good for the agenda setter. As long as we accept the two behavioral assumptions in the second last paragraph in the previous section, this incentive constraint limits nothing. We did so in the joint paper with Dave and Baron and Diermeier (QJE 2001). #]]

Proposal Strategy. Denote the set of Borel probability measures over $\Delta(G)$ by $\mathfrak{P}(\Delta(G))$. Let $\sigma : \Delta(G) \times \{a\} \to \mathfrak{P}(\Delta(G))$ be the mixed proposal strategy of the agenda setter. Given any default $g \in \Delta(G)$, the probability that the agenda setter proposes (and passes) a policy $g' \in \Delta(G)$ is $\sigma(g'|g,a)$.

We are now ready to define the equilibrium.

Definition 1 *A legislative equilibrium of this political system with agenda setter a is a set of reservation value functions* $U_l : \Delta(G) \times \{a\} \to \mathbb{R}$, $l \in \{1,2,3\}$, *and a policy rule* $\sigma : \Delta(G) \times \{a\} \to \mathfrak{P}(\Delta(G))$, *such that:*

1. *Given* σ, *for all* $l \in \{1,2,3\}$ *and all* $g \in \Delta(G)$,

$$U_l(g,a) = (1-\delta)\, g_l + \delta \sum_{g' \in \cdot(G)} \sigma\left(g'|g,a\right) U_l\left(g',a\right).$$

2. *Given* $\{U_l\}_{l=1}^{3}$, *for all* $g, g^* \in \Delta(G)$, $\sigma(g^*|g,a) > 0$ *only if* g^* *solves*

$$\begin{aligned} \max_{g' \in \Delta(G)} \quad & U_a\left(g',a\right) \\ s.t. \quad & U_i\left(g',a\right) \geq U_a(g,a) \text{ for some } i \neq a. \end{aligned}$$

the arguments to make all notations and functions identical to those in Section 4, where we present a dynamic model with multiple sessions.

Condition 1 defines the reservation value functions. Suppose that in one round of policy-making the current default is g. If no new proposal is passed in this round, the policy g prevails as default. With probability $1-\delta$, the legislators have no chance to revisit this policy issue. Therefore g is the final policy outcome and legislator receives g_l. With probably δ the legislative bargaining continues and, assuming that in the next round the agenda setter plays his equilibrium proposal strategy σ, legislator receives an expected utility of $\sum_{g' \in \cdot(G)} \sigma(g'|g,a) U_l(g',a)$.

Condition 2 requires that a policy is proposed by the agenda setter with strictly positive probability only if it maximizes the expected utility of the agenda setter. Given that in order to pass a bill a proposal has to be approved by a majority, a proposed policy must be such that at least one of the other legislators is weakly better off. This explains the constraint in the maximization problem.

If only pure strategies are played in equilibrium, for any $g \in \Delta(G)$, define $f(g,a) \in \Delta(G)$ as the policy such that $\sigma(f(g,a)|g,a) = 1$. We also refer to f as a policy rule in the context with pure strategies.

With a finite policy space and the possibility of mixed strategies, existence of a legislative equilibrium can be easily established by application of standard fixed point theorem. However, for the purpose of the paper, we only characterize the equilibrium for δ sufficiently close to 1.

3 Results and Implications

We are now ready to state the main result. The proof is relegated to an appendix.

Proposition 1 *For any δ sufficiently close to 1, there exists a unique legislative equilibrium, in which for any $g \in \Delta(G)$ and any $a, l \in \{1,2,3\}$,*

$$f_l(g,a) = \begin{cases} \min\limits_{i \neq a} g_i, & \text{if } l \neq a, \\ G - 2\min\limits_{i \neq a} g_i, & \text{if } l = a. \end{cases}$$

In equilibrium and for any initial default policy g^0 such that $f(g^0,a) \neq g^0$, the agenda setter makes a policy proposal $f(g^0,a)$ once and for all and the proposal is passed.

The result has some immediate implications.

First, although the agenda setter is allowed to make a policy proposal at any time and as frequently as possible during the legislative session, in equilibrium, there is only one round of proposal making and voting. In an environment without uncertainty, the collective decision is made once and for all without any amendment.

Second, although it looks as if the legislators played a one-shot legislative bargaining game with closed rule, the possibility of reconsideration changes the nature of the game and makes the equilibrium policy outcome substantially different from what the equilibrium outcome of a static game. Consider the following numerical example.

EXAMPLE 3.1. Assume that $G = 60$, the initial default policy $g^0 = (30, 20, 10)$, and $a = 1$ is the agenda setter. In a static model with closed rule, the policy outcome would be $(50, 0, 10)$. Legislator 3 is disadvantaged by the default policy the most, and therefore becomes the cheapest coalition partner for the agenda setter. Excluded from the coalition, legislator 2 is fully expropriated since her vote is not needed at all to pass the proposal. The agenda setter leaves legislator 3 just enough benefit to break even and takes the rest of the benefit.

In our setup, however, the agenda setter could never pass the policy $(50, 0, 10)$. To see why, consider what would happen if legislator 3 approved this policy counter-factually. With probability $1 - \delta$ the legislators would not have a chance to revisit the policy issue and therefore $(50, 0, 10)$ would be the final policy outcome. With probability δ, however, the agenda setter would be able to propose a new policy $(60, 0, 0)$, which would be accepted by the fully expropriated legislator 2. This implies that by accepting the policy $(50, 0, 10)$, legislator 3 becomes vulnerable. Foreseeing such an adverse consequence, legislator 3 will always vote against the proposal of $(50, 0, 10)$ even though according to this proposal he does not lose any benefit right away. By similar arguments, we can conclude that legislator 3 will not accept any new policy with which legislator 2 receives strictly less benefit than legislator 3. Therefore, legislator 1 as agenda setter can get at most 40 and pass the policy of $(40, 10, 10)$.Importantly, the possibility to reconsider policies limits agenda control *even in the case where there is a sole agenda setter.* This in marked contrast to agenda control models with sincere voting (McKelvey 1976), where an agenda could achieve any point in the policy space or sophisticated voting (Banks 1980, Shepsle and Weingast 1980) where the set of attainable policies is only limited to the Banks set or the Uncovered Set, respectively.

Note also that our result is very different from the result obtained by Bernheim et al. where the last proposer can capture all or almost all of the benefits. The "power of the last word" disappears once we allow for possible ongoing consideration of policy proposals.

Third, the legislators have indirect preferences over the *distribution* of benefits, although the legislators derive utilities only from the benefits they receive. In Example 3.1, legislator 3 strictly prefers $(40, 10, 10)$ to $(50, 0, 10)$ even though both policies leave him 10 units of benefit. Distribution of benefits matters, because through the evolving default, it affects distribution of bargaining power in the rest of the legislative session.

Fourth, except for the agenda setter, all legislators have preferences towards more egalitarian distribution of benefits. In particular, a legislator without the agenda setting power does not want other legislators to be fully expropriated by the agenda setter. However, this demand for "fairer allocations" result from self-interested legislators who want to improve their long-term bargaining position. It does not depend on primitive preference for fair allocations. In a model of decision-making over legislative procedures this insight may have implications for the existence of minority rights and benefits in legislatures.

Five, as a consequence, the agenda setter has limited ability to expropriate the legislator excluded from his winning coalition. Specifically, the value of proposal power in our model is in general smaller that what is implied by a one-shot legislative bargaining game with closed rule.

Six, depending on the initial default policy and the identity of the agenda setter, the policy outcome can be either full equality, full expropriation by the agenda setter, partial expropriation, or policy inertia. This is illustrated in the next set of examples.

EXAMPLE 3.2. (A) Full equality. Suppose that $a = 1$ and $g^0 = \left(x, \frac{1}{3}G, \frac{2}{3}G - x\right)$, where $x \leq \frac{1}{3}G$, then the equilibrium policy outcome is an egalitarian distribution of benefits, $\left(\frac{1}{3}G, \frac{1}{3}G, \frac{1}{3}G\right)$. (B) Full expropriation. Suppose that $a = 1$ and $g^0 = (x, 0, G - x)$ Then in equilibrium the policy outcome is $(G, 0, 0)$ and the agenda setter captures the entire benefit. (C) Partial expropriation. Suppose that $G = 60$, $a = 1$ and $g^0 = (5, 30, 25)$. Then the equilibrium policy is $(10, 25, 25)$. (D) Policy Inertia. Suppose that $a = 1$ and $g^0 = (G - 2x, x, x)$ for some $x \in \left(\frac{1}{3}G, \frac{1}{2}G\right]$. Then the agenda setter is not able to change the policy.

Seven, in contrast to implications derived from legislative bargaining models in the tradition of Baron and Ferejohn (1989), the agenda setter may *not* be one who receives the most benefit in equilibrium. Moreover, the agenda setter may be the legislator who gets the least amount, as shown in Example 3.2 (C). This, however, happens only if the agenda setter is sufficiently disadvantaged by the status quo. This is consistent with episodes in which parties with insuffficent representation take control of the government. Possible cases include minority and especially caretaker governments.

Finally, that the possibility to reconsider policy may create incentives for agenda-setters to rationally make future proposals that will make him worse off than the current proposal. In this case agenda setters would like to commit to making a proposal only once. The possibility to reconsider policy at any time, however, rules out such commitment. Note that, intuitively, commitment would amount to the credible belief that certain policy areas will never be revisited. It is difficult to see how this can be accomplished in a constitutional fashion in a democracy.

In the next two sections we use the base-line model to derive consequences for different economic policy. We first consider entitlement programs similar to the choice environment considered by Kalandrakis (2004). Next we investigate the model's implications in the context of economic fluctuations. As we show below, the model's equilibrium provides a possible explanation for the "ratchet effect" of public spending in parliamentary democracies (Persson and Tabellini 2004).

4 Application: Dynamics of Entitlement Programs

4.1 The Model

Consider a multi-period extension of the core model. That is, suppose the legislators must select the distributive policy for each period $t = 1, 2, ..., T$. We consider the case where T is either potentially large but finite and where T is infinite. Suppose also that each legislator's preferences over policies are intertemporally separable. Given a sequence of policy, $\{g^t\}_{t=1}^T$, the expected and discounted sum of utility of legislator l is given by

$$E\left[\sum_{t=1}^{\infty} \beta^{t-1} g_l^t\right],$$

where β is a common discount factor. In this section we interpret the distributive policy as a continuing entitlement program. That is, once enacted, a policy is in effect until it is reformed through the political process. In every period the legislators meet in a legislative session of the same structure as discussed above. The initial default in a session in period t is the policy chosen in period $t-1$. In each period one legislator is randomly selected as the agenda setter, who is conferred the power to make a policy proposal at any time and as frequent as possible throughout the session. The political process in every session is identical to that in the core model in Section 2. In the beginning of the first period there is an initial status quo policy g^0.

Default and Status Quo. We refer to a "status quo policy" as the policy that has been enacted at the moment the policy issue is deliberated in a legislative session. We refer to a "default policy" as the policy to be implemented at the end of the session if no new bill is passed on the same policy issue in the rest of the same session. Note that in the existing theoretical literature these two concepts coincide (for example, Baron 1996, Kalandrakis 2004, Fong 2006, Baron, Diermeier and Fong 2007, and Duggan and Kalandrakis 2007) this is not necessarily the case in our model.

Comparison with Existing Literature. Kalandrakis (2004) presents a model closest to ours, with only one critical difference. Kalandrakis (2004) assumes that an agenda setter is restricted to make a policy proposal only once in every legislative session, whereas here we assume that an agenda setter could potentially make proposals more than once in one session. Conceptually, the passage of a bill does not prevent the legislature from revisiting the same policy issue before the bill becomes law. Like Kalandrakis, we characterize stationary Markov perfect equilibria and a comparison of equilibrium policy outcomes shows how the possibility of reconsideration substantially shapes legislation. Specifically, Kalandrakis (2004) constructs an equilibrium in which, in the long run, the legislator that serves as agenda setter takes all the benefit, whereas such policy dynamics do not occur in our model for almost all initial status quo policies.

4.2 Equilibrium Definition

Reservation Value Function. We restrict analysis to cases in which the agenda setter conditions his proposal strategy only on the prevailing default policy. Consider an arbitrary round of bargaining in period t with agenda setter a and default $g \in \Delta(G)$. Let $U_l(g, a, t)$ be legislator l's reservation value with default g in period t. This is the expected utility of legislator l if no new policy is passed in the current legislative period.

Continuation Value Function. Let $V_l(g, t)$ be legislator l's continuation value with a status quo policy g in period t before the agenda setter is randomly selected.

Proposal Strategy. Again we model voting by an incentive compatibility constraint and only explicity formulate the proposal strategy of an agenda setter. Let $\sigma : \Delta(G) \times \{1,2,3\} \times \{1,...,T\} \to \mathfrak{P}(\Delta(G))$ be the mixed proposal strategy of the respective agenda setter. In particular, given any default $g \in \Delta(G)$, and any period t, the probability that legislator a as agenda setter proposes (and passes) a policy $g^* \in \Delta(G)$ is $\sigma(g^*|g,a,t)$.

We are now ready to define the equilibrium concept [[Pohan, why do we drop Markovian here? It still applies in each period, no? **Pohan's Answer:** Yes and No. We still retain the Markovian structure *within* every session with respect to policy making with evolving default policies. However, *across sessions* this is a finite-horizon game with T periods, and we define a subgame perfect equilibrium.In the infinite-horizon game in the next subsection, we have Markovian structures both within and across legislative sessions. Will add more words to explain it in the paper later.#]].

Definition 2 *A subgame perfect equilibrium of this political system with a finite horizon is a set of reservation value functions, $U_l : \Delta(G) \times \{1,2,3\} \times \{1,...,T\} \to \mathbb{R}$, $l \in \{1,2,3\}$, a set of continuation value functions, $V_l : \Delta(G) \times \{1,...,T\} \to \mathbb{R}$, $l \in \{1,2,3\}$, and a policy rule $\sigma : \Delta(G) \times \{1,2,3\} \times \{1,...,T\} \to \mathfrak{P}(\Delta(G))$, such that:*

1. *For all $l \in \{1,2,3\}$ and all $g \in \Delta(G)$, $V_l(g, T+1) = 0$.*

2. *Given f, for all $t \in \{1,...,T\}$, all $l \in \{1,2,3\}$ and all $g \in \Delta(G)$,*

$$V_l(g,t) = \frac{1}{3} \sum_{a \in \{1,2,3\}} \sum_{g^* \in \Delta(G)} \sigma(g^*|g,a,t) \left[(1-\beta) g_l^* + \beta V_l(g^*, t+1)\right].$$

3. *Given f and $\{V_l\}_{l=1}^3$, for all $t \in \{1,...,T\}$, all $a, l \in \{1,2,3\}$ and all $g \in \Delta(G)$,*

$$U_l(g,a,t) = (1-\delta)((1-\beta) g_l + \beta V_l(g,t+1)) + \delta \sum_{g^* \in \Delta(G)} \sigma(g^*|g,a,t) \left[(1-\beta) g_l^* + \beta V_l(g^*, t+1)\right].$$

4. *Given $\{U_l\}_{l=1}^3$, for all $t \in \{1,...,T\}$, all $a \in \{1,2,3\}$ and all $g, g^* \in \Delta(G)$, $\sigma(g^*|g,a) > 0$ only if g^* solves*

$$\max_{g' \in \Delta(G)} U_a(g',a,t)$$
$$s.t. \quad U_i(g',a,t) \geq U_i(g,a,t) \text{ for some } i \neq a.$$

This definition of equilibrium needs some explanation.

If only pure strategies are played in equilibrium, for any $g \in \Delta(G)$, define $f(g,a,t) \in \Delta(G)$ as the policy such that $\sigma(f(g,a,t)|g,a,t) = 1$. We also refer to f as a "policy rule" in the case of pure strategies.

With a finite policy space and the possibility of mixed strategies, existence of a legislative equilibrium can be easily established. However, as above, for the purpose of the paper, we only characterize the equilibrium for δ sufficiently close to 1.

4.3 Analysis

This model can be solved by backward induction. The last period is isomorphic to the core model with one single legislative session. Now consider the penultimate period. A policy choice in period $T-1$ affects not only a legislator's instantaneous utility in that period but also continuation value in period T through the moving status quo. However, we claim that in this model an agenda setter does not strategically manipulate the status quo. To see this, consider the following thought experiment.

[[Pohan, please check the next sentences carefully]]. [[Consider the case where the legislators bargain[[-]] over the default policy for period T before a new agenda settter is randomly selected. Suppose that legislator 1 is the period-$T-1$ agenda setter and he has to the power to make a proposal of a default policy with the proposal being put to a vote against the current default, i.e. the policy chosen in the previous period. A simple backward induction calculation shows that in equilibrium the chosen initial default for period T gives the legislator 1 a benefit of $\frac{1}{2}G$, some other legislator (say, legislator 2) also $\frac{1}{2}G$, and the third legislator 0. [[Pohan, does this depend on anything or is it default independent. **Pohan's Answer:** This is default independent. Because an expected utility of $\frac{1}{2}G$ is the globally maximal continuation value any legislator would have. Come back to this later.#]]. This default policy $\left(\frac{1}{2}G, \frac{1}{2}G, 0\right)$ leaves both legislators 1 and 2 a unconstrained maximal continuation value of $\frac{1}{2}G$, and therefore once proposed, it would be supported by 1 and 2 in the majority voting.

Suppose then that in period $T-1$ the initial default is $\left(\frac{1}{6}G, \frac{1}{3}G, \frac{1}{2}G\right)$ and keep in mind that legislator 1 is the agenda setter. Could he propose and pass a policy of $\left(\frac{1}{2}G, \frac{1}{2}G, 0\right)$? At the first glance, this proposal gives legislator 1 a maximal continuation value in the last period and higher current utility that what the default gives him. However, $\left(\frac{1}{2}G, \frac{1}{2}G, 0\right)$

will never be a policy outcome in period $T-1$. This is because once a bill of $\left(\frac{1}{2}G, \frac{1}{2}G, 0\right)$ is passed, it becomes the default in the rest of the legislative session in $T-1$. With probability δ, which is assumed to be close to 1, the agenda setter gets a chance to revisit the policy and he has a strong incentive change this policy. Should $\left(\frac{1}{2}G, \frac{1}{2}G, 0\right)$ be the default, legislator 1 as agenda setter would like to proposal and pass a policy of $(G, 0, 0)$, which would be supported by legislator 3.Therefore the proposal of $\left(\frac{1}{2}G, \frac{1}{2}G, 0\right)$ would not be approved by voter 2.

If $\left(\frac{1}{2}G, \frac{1}{2}G, 0\right)$ is the policy outcome in period $T-1$, legislator 1 has a continuation value of $\frac{1}{2}G$ in the final period. Therefore, in the last two periods legislator 1 receives a total utility of

$$\left(\tfrac{1}{2}G\right) + \beta\left(\tfrac{1}{2}G\right) = (1+\beta)\left(\tfrac{1}{2}G\right).$$

If $(G, 0, 0)$ is the policy outcome in period $T-1$, legislator 1 has a continuation value of $\frac{1}{3}G$ in the final period. Therefore, in the last two periods legislator 1 receives a total utility of

$$(1-\beta)\left[G + \beta\left(\tfrac{1}{3}G\right)\right] = (1-\beta)\left(1+\tfrac{1}{3}\beta\right)G.$$

For any $\beta \in [0, 1)$, the policy $(G, 0, 0)$ gives legislator 1 a larger total utility in the last two periods. This implies that $\left(\frac{1}{2}G, \frac{1}{2}G, 0\right)$ cannot be the final policy outome in period $T-1$. Foreseeing that legislator 1 as agenda setter will not finalize the policy choice at $\left(\frac{1}{2}G, \frac{1}{2}G, 0\right)$, legislator 2 will not accept this proposal. This eliminates strategic manipulation of the status quo by any agenda setter. It is straightforward to show that in period $T-1$ an agenda setter makes the same policy proposal as in a one-sesseion game.

Repeating this argument by backward inductions yields the next proposition.

Proposition 2 *Suppose that T is finite. For any δ sufficiently close to 1, there exists a unique symmetric subgame perfect equilibrium, in which for any $t \in \{1, ..., T\}$, any $g^{t-1} \in \Delta(G)$, and any $a, l \in \{1, 2, 3\}$,*

$$f_l(g, a, t) = \begin{cases} \min\limits_{i \neq a} g_i, & \text{if } l \neq a, \\ 1 - 2\min\limits_{i \neq a} g_i, & \text{if } l = a. \end{cases}$$

4.4 The Infinite Horizon Model

Again, we restrict analysis to cases in which the agenda setter conditions his proposal strategy only on the prevailing default. Consider an arbitrary round of negotiation in an arbitrary period with agenda setter a and default g. Let $U_l^*(g,a)$ be legislator l's reservation value with default g. This corresponds to the expected utility of legislator l if no new policy is passed in this round of negotiation in the current period. Let $V_l^*(g)$ be legislator l's continuation value with a status quo policy g in the beginning of the period before an agenda setter is randomly selected. Again we model voting by an incentive compatibility constraint and only explicity formulate the proposal strategy of an agenda setter. Let $\sigma^* : \Delta(G) \times \{1,2,3\} \to \mathfrak{P}(\Delta(G))$ be the mixed proposal strategy of the agenda setter. In particular, given any default g and any period t, the probability that legislator a as agenda setter proposes (and passes) a policy $g^* \in \Delta(G)$ is $\sigma^*(g^*|g,a)$.

We are ready to define the equilibrium in the infinite-horizon model.

Definition 3 *A stationary Markov perfect equilibrium of this political system with an infinite horizon is a set of reservation value functions,* $U_l^* : \Delta(G) \times \{1,2,3\} \to \mathbb{R}$, $l \in \{1,2,3\}$, *a set of continuation value functions,* $V_l^* : \Delta(G) \to \mathbb{R}$, $l \in \{1,2,3\}$, *and a policy rule* $\sigma^* : \Delta(G) \times \{1,2,3\} \to \mathfrak{P}(\Delta(G))$, *such that:*

1. *Given* σ^*, *for all* $l \in \{1,2,3\}$ *and all* $g \in \Delta(G)$,

$$V_l^*(g) = \frac{1}{3} \sum_{a \in \{1,2,3\}} \sum_{g^* \in \Delta(G)} \sigma^*(g^*|g,a)\left[(1-\beta) g_l^* + \beta V_l^*(g^*)\right].$$

2. *Given* σ^* *and* $\{V_l^*\}_{l=1}^3$, *for all* $a, l \in \{1,2,3\}$ *and all* $g \in \Delta(G)$,

$$U_l^*(g,a) = (1-\delta)\left((1-\beta) g_l + \beta V_l^*(g)\right) + \delta \sum_{g^* \in \Delta(G)} \sigma^*(g^*|g,a)\left[(1-\beta) g_l^* + \beta V_l^*(g^*)\right].$$

3. *Given* $\{U_l^*\}_{l=1}^3$, *for all* $a \in \{1,2,3\}$ *and all* $g, g^* \in \Delta(G)$, $\sigma^*(g^*|g,a) > 0$ *only if* g^* *solves*

$$\begin{aligned} \max_{g' \in \Delta(G)} \quad & U_a^*(g',a) \\ s.t. \quad & U_i^*(g',a) \geq U_i^*(g,a) \text{ for some } i \neq a. \end{aligned}$$

The definition of the equilibrium needs some explanation. There is a tricky recursive fixed-point problem. [[Pohan, can you add some more? **Pohan's Response:** I haven't had time adding more words. Will do later.]]

As above, if only pure strategies are played in equilibrium, for any $g \in \Delta(G)$, define $f(g,a) \in \Delta(G)$ as the policy such that $\sigma^*(f^*(g,a)|g,a) = 1$. We again refer to f^* as a policy rule in the context with pure strategies.

Similar to the previous cases, existence of a legislative equilibrium can be easily established. However, for the purpose of the paper, we only characterize the equilibrium for δ sufficiently close to 1. The next proposition presents an infinite-horizon equilibrium that is the limit of finite-horizon subgame perfect equilibria as time horizon T goes to infinity.

Proposition 3 ([Consider the infinite horizon model.]) *For any δ sufficiently close to* 1, *there exists a stationary Markov perfect equilibrium, in which for any $g \in \Delta(G)$, and any $a, l \in \{1, 2, 3\}$,*

$$f_l(g, a, t) = \begin{cases} \min\limits_{i \neq a} g_i, & \text{if } l \neq a, \\ G - 2\min\limits_{i \neq a} g_i, & \text{if } l = a. \end{cases}$$

Along this equilibrium path, policy dynamics in the long run depends on the initial status quo policy g^0 and identity of the first agenda setter a_1. If $g^0_{a_1} \geq \min_{i \neq a_1} g^0_i$ or $g^0_{a_1} < \min_{i \neq a_1} g^0_i \leq \frac{1}{3}$, in any period, the agenda setter gets $G - 2\min_{i \neq a_1} g^0_i$, and each of the other legislators gets $\min_{i \neq a_1} g^0_i$. If $\max\left\{g^0_{a_1}, \frac{1}{3}\right\} < \min_{i \neq a_1} g^0_i$, then until any $i \neq a_1$ obtains power for the first time, in every period a_1 as agenda setter gets $G - 2\min_{i \neq a_1} g^0_i$, which is strictly less than $\min_{i \neq a_1} g^0_i$, what each of the other legislators gets; after the first time the initial agenda setter loses power, in every period the agenda setter $4\min_{i \neq a_1} g^0_i - G$, which is strictly greather than $G - 2\min_{i \neq a_1} g^0_i$, what each of the other legislators gets.

4.5 Interpretation and Discussion]

First, similar to what is in the core model with one single legislative session, although an agenda setter can possibly propose a new policy at any time when holding power, he makes a proposal and makes a new policy, if any, immediately after obtaining power. Without any uncertainty other than stochastic turnover of power, the policy does not change as long as the agenda setter does not change. However, the fact that every agenda setter makes a new policy choice once and for all does not imply that he is restricted to do so. Rather, this effect occurs in equilibrium. With frequent possibilities of collective decision makings, the policy outcome is substantially different from a setup in which every agenda setter is constrained to make a new policy only once like in, for example, the model of Kalandrakis (2004).

Second, note that the policy an agenda setter chooses in a multiperiod setup is equal to what he would choose in a one-session setup. There is no manipulation of the status quo by any agenda setter.

Third, the model implies very rich policy dynamics. Depending on the initial status quo policy and the identity of the first agenda setter, in the long run there are three different patterns of policy dynamics.

1. Full Expropriation by Any Agenda Setter

After finite periods of transition, whoever is the agenda setter takes all and leaves nothing to the others. Illustrated by Example 4.1, such policy dynamics is reminiscent of Kalandrakis (2004). However, in our model, full expropriation happens only if the initial status quo is so unequal that some legislator originally gets nothing.

Example 4.1. $G = 60$, $g^0 = (30, 30, 0)$

time period	first	second	third	fourth	...
agenda setter	3	3	1	2	...
policy outcome	$(30, 30, 0)$	$(30, 30, 0)$	$(60, 0, 0)$	$(0, 60, 0)$	...

2. Inequality-Inclined Allocation of Benefit

After finite periods of transition, in every period the agenda setter receives more benefits than the others, although he does not capture all. An agenda setter therefore has limited ability to expropriate the legislator excluded from his winning coalition. The legislators without proposal power receive equal shares. This is illustrated by Examples 4.2 and 4.3.

Example 4.2. $G = 60$, $g^0 = (25, 25, 10)$

time period	first	second	third	fourth	...
agenda setter	3	3	1	2	...
policy outcome	$(25, 25, 10)$	$(25, 25, 10)$	$(40, 10, 10)$	$(10, 40, 10)$	...

Example 4.3. $G = 60$, $g^0 = (30, 20, 10)$

time period	first	second	third	fourth	...
agenda setter	2	3	1	2	...
policy outcome	$(10, 40, 10)$	$(10, 10, 40)$	$(40, 10, 10)$	$(10, 40, 10)$	...

3. Full Egalitarian Distribution of Benefits

The policy converges to a fully egalitarian distribution of benefits right away, and thereafter no agenda setter is able to change it anymore. All agenda setters, except for the first one, have no advantage with power. This is illustrated by Example 4.4.

Example 4.4. $G = 60$, $g^0 = (30, 20, 10)$					
time period	first	second	third	fourth	...
agenda setter	3	2	1	2	...
policy outcome	$(20, 20, 20)$	$(20, 20, 20)$	$(20, 20, 20)$	$(20, 20, 20)$	...

Finally, as shown in Examples 4.3 and 4.4 with the same initial status quo, policy dynamics in the long run is path-dependent. In particular, it not only depends on the initial default, but also who obtains power in the first period. This model thus provides an example of how political players in an early stage of democracy and the initial distribution of benefit may affect the long-run pattern of fiscal policy.

5 Application: The Ratchet Effect of Government Spending

5.1 Stylized Facts and General Ideas

Recent empirical studies by Persson and Tabellini show that, in parliamentary countries with proportional representation, government spending as a fraction of GDP goes up during cyclical downturns but does not come down during cyclical upturns, whereas this "ratchet effect" is not apparent in countries with other constitutional arrangements. This section presents a simple political economy model based on the analytical framework in Section 2 and shows how the legislative institutions typical of parliamentary democracies may lead to the asymmetric movements of government spending.

The observation of the ratchet effect is due to Persson and Tabellini (2001, 2003, and 2004). They divide democratic countries into four constitutional groups, and empirically investigate how different constitutional arrangements shape fiscal policies. They show that proportional-parliamentary democracies differ from all other groups – majoritarian-presidential, majoritarian-parliamentary, and proportional-presidential – in terms of fiscal policy dynamics.

First, government expenditure, fiscal deficit and welfare spending are more persistent in this group than in the others. Second, spending as a percentage of GDP increases in cyclical down-turns but does not decrease in booms. In other words, downturns lead to a lasting expansion of outlays and welfare spending in proportion to GDP that is not reversed during upturns. Such *ratchet effect of government spending* is absent in all other constitutional groups. Third, the difference in the size of government between this group and the others grew particularly large in the period up to the early 1980s (or the early 1990s in the case of welfare spending).

What makes proportional-parliamentary democracies so *special*? Proportional representation leads to minority parliaments; i.e. no party obtains a majority of seats in parliament. This is true even if voters can vote strategically (Baron and Diermeier (2001) and if governments can strategically manipulate future status quos (Fong 2006, Baron, Diermeier, and Fong 2007). Therefore government policy needs to be conceptualized as bargaining among multiple parties, either among *all* parties represented in the parliament or among the parties represented in the governing coalition (Diermeier and Feddersen 1998). Note that this feature is absent in all other constitutional groups. For example, parliamentary democracies with plurality rule (e.g. the UK) encourage two major political parties. Except for the rare case of a hung parliament, the party who controls a majority of seats usually has full control over policy. One the other hand, presidential democracies (whether multi-party or two-party) lack the constitutional feature of effective agenda control by the executive. So, our model combines the features typical of parliamentary democracies (the government's agenda control) with multi-party bargaining. In the context of a simple model we show below how the friction resulting from multilateral bargaining is the key to the distinguished patterns of fiscal policy dynamics.

The intuition is as follows. A sizable fraction of total government expenditure is related to entitlement programs. In those programs benefits are distributed. Moreover, once enacted, they are in effect until they are reformed. When an economy is hit by a temporary negative income shock, the party that controls agenda setting faces a strong resistance on expenditure cuts. This is because a more stringent entitlement program on any socioeconomic group implies a worse status quo in the future and therefore a permanently lower bargaining power of that group. Fiscal adjustment in response to a temporary shock has a permanent effect. This makes it extremely difficult for an agenda setter, whose power is persistent and may last for a certain amount of time, to cut down expenditures on the other groups. On the other hand, with a temporary positive income shock, the leading party can easily satisfy its coalition partners by their reservation values and pass a more generous entitlement program to benefit the socioeconomic group it represents. An asymmetric movement of public spending thus results.

5.2 A Simple Model: Impulse Response Analysis

In this section we present a dynamic model similar to that in Section 4 with one twist. The total size of benefit, or the total government expenditure, is endogenous. Imagine that every period the three legislators divide a pie but the size of the pie is not fixed. They have to jointly produce a total benefit of $G \in \mathbb{N}$ and then divide it. A policy is therefore a vector (g_1, g_2, g_3, G), where $G = \sum_{l=1}^{3} g_l$. Public production is costly and parties equally share the cost. The cost function depends on the state of the economy denoted by s. For simplicity, we assume that the cost function is piecewise linear:

$$C(G, s) = \begin{cases} 0, & \text{if } G \leq \overline{G}_s, \\ (1+c)\left(G - \overline{G}_s\right), & \text{if } G > \overline{G}_s, \end{cases}$$

for some $c > 0$. Assume that $s \in \{H, N, L\}$, where H stands for high, N for normal, and L for low. In good states, the marginal cost is smaller; In bad states, the marginal cost is larger. Therefore we assume that $\overline{G}_H > \overline{G}_N > \overline{G}_L$. We do not explicitly model income shocks. However, we conjecture that in this simple framework the effect of a negative public production shock should be similar to that of a negative income shock in a full-fledged public finance model.

The preference of legislator l is represented by

$$E\left[\sum_{t=0}^{\infty} \beta^t \left(g_l^t - \tfrac{1}{3} C\left(G^t, s^t\right)\right)\right],$$

where $\beta \in [0, 1)$ is a common discount factor as before.

As a normative benchmark, in the first best, every period the total size of government expenditure is equal to $\overline{G}_s$; public spending is fully responsive to state of the economy. In our model, the policy is chosen by the political process of legislative bargaining. We want to show how the equilibrium policy deviates from the first best and identify a possible source of bargaining frictions.

Instead of fully characterizing this model with a general stochastic process that governs transition of states, we only report an impulse response experiment in this section. We assume that for every period, the state is stable and normal, and in period 1, the economy is hit by an unexpected temporary shock; $s^1 \in \{H, L\}$. The exact interpretation of this shock is not critical. What really matters is that the economy temporarily deviates from its long-run trend; we now want to investigate how this fluctuation results in fluctuation of government spending. Moreover, we assume that power is persistent and legislator 1 is the agenda setter for every period. To obtain our result we only require some degree of power persistence. That is, the party who will control agenda setting in the next quarter is very likely to be the party currently in power. In what follows, we divide the discussion into cases with a positive shock, $s^1 = H$, and a negative shock, $s^1 = H$. We assume that the initial status quo policy is such that the total spending $G^0 = \overline{G}_N$ and $g_2^0 \geq g_3^0$.

Case I: A Temporary Positive Shock.

In period 1 with $s^1 = H$, it is as if the agenda setter chooses a policy $\left(g_1^1, g_2^1, g_3^1, G^1\right) \in \mathbb{Z}_+^4$ in order to maximize

$$(1-\beta)\left(g_1^1 - \frac{1}{3}(1+c)\max\left\{0, G^1 - \overline{G}_H\right\}\right) + \beta\left(\overline{G}_N - 2g_3^1\right),$$

subject to

$$g_1^1 = G^1 - g_2^1 - g_3^1,$$

$$(1-\beta)\left(g_3^1 - \frac{1}{3}(1+c)\max\left\{0, G^1 - \overline{G}_H\right\}\right) + \beta g_3^1 \geq (1-\beta)\, g_3^0 + \beta g_3^0,$$

and

$$\begin{aligned} & (1-\beta)\left(g_3^1 - \frac{1}{3}(1+c)\max\left\{0, G^1 - \overline{G}_H\right\}\right) + \beta g_3^1 \\ = \; & (1-\beta)\left(g_2^1 - \frac{1}{3}(1+c)\max\left\{0, G^1 - \overline{G}_H\right\}\right) + \beta g_2^1. \end{aligned}$$

This equivalent maximization problem needs some explanation. Given any period-one policy choice $g^1 = \left(g_1^1, g_2^1, g_3^1\right)$, the continuation values of all three legislators from the second period are $\overline{G}_N - 2g_3^1$, g_3^1, g_3^1 respectively. The first constraint is the resource constraint (or balanced budget constraint). According to what we learned from Sections 2 to 4, the agenda setter makes a policy proposal that satisfies one of the other legislators with the lowest reservation value. In our example, this is legislator 3 and this explains the second constraint. Finally, in order for legislator 3 to accept the proposal, it has to be the case that legislator 2 does not receive a lower utility level than him. Otherwise, in the rest of the session in period 1, the agenda setter would revisit the policy and propose another policy that seeks support from legislator 2. The last constraint summarizes this equal-utility property, which is equivalent to the constraint that $g_2^1 = g_3^1$.

In equilibrium, (A) $g_2^t = g_3^t = \min\{g_2^0, g_3^0\}$ for all t, (B) $G^1 = \overline{G}_H$ and $G^t = \overline{G}_N$ for all $t > 1$, and (C) $g_1^1 = \overline{G}_H - 2\min\{g_2^0, g_3^0\}$ and $g_1^t = \overline{G}_N - 2\min\{g_2^0, g_3^0\}$.

With a temporary positive shock, total expenditure expands accordingly, and all extra spending goes to the party that controls the agenda. The legislators without power do not benefit from the positive shock. After the shock, total spending is back to its normal level.

Case II: A Temporary Negative Shock.

In period 1 with $s^1 = L$, it is as if the agenda setter chooses a policy $\left(g_1^1, g_2^1, g_3^1, G^1\right) \in \mathbb{Z}_+^4$ in order to maximize

$$(1-\beta)\left(g_1^1 - \frac{1}{3}(1+c)\max\left\{0, G^1 - \overline{G}_L\right\}\right) + \beta\left(\overline{G}_N - 2g_3^1\right),$$

subject to

$$g_1^1 = G^1 - g_2^1 - g_3^1,$$

$$(1-\beta)\left[g_3^1 - \tfrac{1}{3}(1+c)\max\left\{0, G^1 - \overline{G}_L\right\}\right] + \beta g_3^1 \geq (1-\beta)\left[g_3^0 - \tfrac{1}{3}(1+c)\left(\overline{G}_N - \overline{G}_L\right)\right] + \beta g_3^0,$$

and

$$\begin{aligned} & (1-\beta)\left(g_3^1 - \frac{1}{3}(1+c)\max\left\{0, G^1 - \overline{G}_L\right\}\right) + \beta g_3^1 \\ = \; & (1-\beta)\left(g_2^1 - \frac{1}{3}(1+c)\max\left\{0, G^1 - \overline{G}_L\right\}\right) + \beta g_2^1. \end{aligned}$$

The second constraint guarantees that legislator 3, the "cheaper" possible coalition partner, is indifferent to the new period-1 policy. The last constraint requires that legislator 2, the one excluded from the winning coalition, is offered the same total utility as legislator 3. This is equivalent to the constraint that $g_2^1 = g_3^1$. Combining the last two conditions we have

$$g_2^1 = g_3^1 = \widehat{g} \equiv g_3^0 + \tfrac{1}{3}(1-\beta)(1+c)\max\left\{0, G^1 - \overline{G}_N\right\}.$$

Substituting the binding constraints, the agenda setter's objective function is simplified to

$$\begin{aligned} & (1-\beta)\left\{G^1 - 2\left[g_3^0 - \tfrac{1}{3}(1-\beta)(1+c)\max\left\{0, G^1 - \overline{G}_N\right\}\right] - \tfrac{1}{3}(1+c)\max\left\{0, G^1 - \overline{G}_L\right\}\right\} \\ & +\beta\left\{\overline{G}_N - 2\left[g_3^0 - \tfrac{1}{3}(1-\beta)(1+c)\max\left\{0, G^1 - \overline{G}_N\right\}\right]\right\}. \end{aligned}$$

There is only one unknown G^1 in the objective function. It can be verified that every $G^1 > G_N$ makes the agenda setter worse off than $G^1 = G_N$, so the equilibrium period-1 total spending must be no more than $\overline{G}_N$. Moreover, $G^1 \geq \overline{G}_L$. Given these, the objective function is simplified to

$$-(1-\beta)\, c\, G^1 + \textit{constant}.$$

Therefore, the agenda setter chooses G^1 as small as possible given that that $G^1 \geq \overline{G}_L$ and the incentive constraints for legislator 3 to accept the policy.

In equilibrium, if $2\widehat{g} \leq \overline{G}_L$, i.e., $\overline{G}_L$ is sufficiently large, we have $G^1 = \overline{G}_L$. In this case, $g_2^1 = g_3^1 = \widehat{g}$ and $g_1^1 = \overline{G}_L - 2\widehat{g}$. If the negative shock is not too bad, total spending is fully adjusted downward to its socially optimal level. The spending on the legislators without power are only slightly cut down. With reasonable parameter values, g_2^1 and g_3^1 are very close to $\min\left\{g_2^0, g_3^0\right\}$. Most of the fiscal adjustment is done by cutting the spending that benefits the agenda setter.

In equilibrium, if $2\widehat{g} > \overline{G}_L$, i.e., $\overline{G}_L$ is sufficiently small, we have $G^1 = 2\widehat{g} > \overline{G}_L$. In this case, $g_2^1 = g_3^1 = \widehat{g}$ and $g_1^1 = 0$. Now, if the negative shock is very severe, total spending cannot be fully adjusted downward to its socially optimal level. There is overspending in the bad state. The spending on the legislators without power are only slightly cut down, whereas the spending on the agenda setter is totally cut down to zero. The fact that the agenda setter is receiving zero benefit in period 1 cannot be interpreted literally. If the agenda setter's marginal utility of his benefit becomes sufficiently large as spending on him is cut, in equilibrium, g_1^1 may be strictly positive even if the bad shock is very severe.

This simple model with an impulse analysis yields various testable empirical implications. Importantly it generates a version of the ratchet effect of total spending. As the economy is hit by an unexpected temporary positive shock, the total spending expands and all extra spending goes to the agenda setter. As the economy is hit by an unexpected temporary negative shock, the total spending may not be fully downward adjusted. Whether it does depends on the size of the negative shock. The intuition is that the agenda setter has difficulty cutting spending on the other legislators; he can mainly cut down his own benefit. If the negative shock is small, the agenda setter is able to do so and adjust total spending to its new socially optimal value. However, if the negative shock is sufficiently large, the agenda setter leaves zero benefit to himself and at this corner solution, he is not able to further reduce public spending. As a consequence, there is overspending compared to the first best solution.

Our model also raises some new empirical questions. First, in the empirical studies

Persson and Tabellini identify the ratchet effect in parliamentary democracies with proportional representation. Our intuition suggests that what matters is the form of governance that makes fiscal policy decisions. That is, whether or not the fiscal policy is a bargaining outcome by multiple parties. To test this tuition, we could possibly look at fiscal policy dynamics in those countries during different regimes: regimes with a majority party, and regimes with a minority parliament and coalition governments. We conjecture that the ratchet effect is more prevalent when fiscal policy is determined by a coalition government. Second, ratchet effects are particularly pronounced for large negative shocks. Third, it is worth looking at the composition of government spending. In particular, over the business cycles, how does spending on different items fluctuate? Our model implies that in a bad state, the agenda setter's spending is cut down the most. Whether these predictions are observed in the data is an open question.

6 Concluding Remarks

This paper proposes a new analytical framework of legislative bargaining. The setup, we believe, captures a few realistic features in legislatures, for example, real-time legislation, persistent agenda setting power, possibility of reconsideration without a well-defined last round of negotiation. These features are particularly characteristic of parliamentary democracies. The model is tractable and can be easily applied in dynamic models, to provide a positive analysis of fiscal policy.

An immediate next step of this research agenda is to extend the core model to include an arbitrary number of legislators with any decision rule and fully characterize all legislative equilibria. We think the same intuitions carry over to a more general context, including the key insight that an agenda setter has limited ability to expropriate legislators excluded from the winning coalition if he holds the power to reconsider the policy issue at some later dates. It also would be interesting to investigate how the value of agenda setting and distribution of benefits are affected by the size of legislature and voting rules.

Another important extension of the core model is to replace the agenda setter by a gatekeeper. We define a gatekeeper as the legislator who is conferred the veto right to block any policy proposal made by some others and at the same time able to propose a new policy in some situations. The sequence of events in the game can be modified as follows: There is an initial default and one legislator is randomly assigned to be a gatekeeper. The legislators then are able to make policy proposals in turn. A legislator can choose to pass his turn if proposing a policy does not make himself better off. Once a proposal is made, it has to be approved by the gatekeeper and then voted on against the default by majority rule. A passed proposal becomes the new default in future rounds of negotiation. Legislative interaction ceases after all legislators pass. The final default policy is implemented. In a model like this, we would be able to compare the respective value of proposal and veto power.

Finally, this analytical framework could be incorporated into fuller developed models of public finance and macroeconomic policy choice. As recent empirical studies on political economy and comparative constitutions have established more stylized facts and raise new questions about how political institutions shape dynamics of policy, we expect fruitful developments from this approach.

Appendix (Very incomplete!!)

Proof of Proposition 1..

STEP 1. Initially conjecture that in every period the agenda setter makes a proposal according to the pure-strategy policy rule f as specified in the Proposition. With this conjectured policy rule, construct the value functions for all players according to Condition 1 of Definition 1. Denote the constructed value function for player $l \in \{1, 2, 3\}$ by $\widehat{U}_l$. The goal is to solve the following nonlinear functional equation system:

$$\widehat{U}_l(g) = (1-\beta) g_l + \beta \widehat{U}_l(f(g,a)), \quad \text{for all } l \in \{1,2,3\}.$$

We solve this system by a series of claims.[4]

CLAIM 1.1. For any $g \in \Delta(G)$ such that $g_j = g_k$ for all $j.k \neq a$, $f(g,a) = g$ and therefore $\widehat{U}_l(g) = (1-\beta) g_l + \beta \widehat{U}_l(g) = g_l$, for all $l \in \{1,2,3\}$.

CLAIM 1.2. For any $g \in \Delta(G)$, since $f_j(g,a) = f_k(g,a)$ for all $j,k \neq a$, by Claim 1.1, $\widehat{U}_l(f(g,a)) = f_l(g,a)$, for all $l \in \{1,2,3\}$.

CLAIM 1.3. Implied by Claim 1.2 and the conjectured policy rule,

$$\widehat{U}_l(g) = \begin{cases} (1-\beta) g_i + \beta \min\limits_{i \neq a} g_i, & \text{if } l \neq a, \\ (1-\beta) g_a + \beta \left(G - 2 \min\limits_{i \neq a} g_i \right), & \text{if } l = a. \end{cases} \tag{1}$$

The conjectured value functions have a natural interpretation: If a party believes that in all subsequent periods the policy will evolve following the policy rule f, $\left\{\widehat{U}_i\right\}_i$ are then the value functions he perceives.

STEP 2. Given the value functions constructed in Step 1, we now show that the conjectured policy rule f satisfies Condition 2 of Definition 1. Throughout this step of the proof, take any $g \in \Delta(G)$. Also take any $g^* \in \Delta(G)$ that maximizes $U_a(g)$ subject to the constraint that $\widehat{U}_i(g^*) \geq \widehat{U}_i(g)$ for some $i \neq a$. With a series of claims we characterize necessary conditions for g^*.

PROOF OF CLAIM 2.1. Suppose there exists $g' \in \Delta(G)$ such that $\min_{i \neq a} g'_i < \min_{i \in \neq a} g_i$ and $\widehat{U}_j(g') \geq \widehat{U}_j(g)$ for some $j \neq a$ and for all d sufficiently small. Then

$$\widehat{U}_j(g') - \widehat{U}_j(g) = (1-\beta)\left(g'_j - g_j\right) + \beta\left(\min\nolimits_{i \neq a} g'_i - \min\nolimits_{i \neq a} g_i\right), \quad \text{and}$$
$$\lim\nolimits_{d \to 0}\left[\widehat{U}_j(g') - \widehat{U}_j(g)\right] = \min\nolimits_{i \neq a} g'_i - \min\nolimits_{i \neq a} g_i < 0.$$

[4] Claims without proofs are obvious ones.

This is a contradiction. Since the agenda setter has to satisfy at least one player in $\{1,2,3\}_a$ by his reservation value, it must be that $\min_{i\neq a} g'_i \geq \min_{i\neq a} g_i$.

CLAIM 2.2. For all $j,k\neq a$, if $\widehat{U}_j(g^*)\geq \widehat{U}_j(g)$, then $g^*_j\geq g^*_k$.

PROOF OF CLAIM 2.2. Suppose $g^*_k > g^*_j$ and consider $g''\in\Delta(G)$ such that $g''_a = g^*_a+\varepsilon$, $g''_j = g^*_j$ and $g''_k = g^*_k-\varepsilon$ for some $\varepsilon\in\left(0,\min\left\{g^*_k-g^*_j, G-g^*_a\right\}\right)$. Then

$$\widehat{U}_j(g'') = g^*_j = \widehat{U}_j(g^*)\geq \widehat{U}_j(g), \quad \text{and}$$
$$\widehat{U}_a(g'')-\widehat{U}_a(g^*) = (1-\beta)\varepsilon > 0.$$

This is a contradiction.

CLAIM 2.3. Let $j,k\neq a$ be distinct players. If $g_j = g_k$, then for all d sufficiently small, $g^*_j = g^*_k$.

PROOF OF CLAIM 2.3. Without loss of generality, suppose $\widehat{U}_j(g^*)\geq \widehat{U}_j(g)$. By Claims 2.1 and 2.2, $g^*_j\geq g^*_k\geq g_k$. Therefore, $\widehat{U}_k(g^*)\geq\widehat{U}_k(g)$. Since $\widehat{U}_i(g^*)\geq \widehat{U}_i(g)$ for all $i=j,k$, by Claim 2.2, $g^*_j = g^*_k$.

CLAIM 2.4. Let $i,j\neq a$ be distinct players. If $g_j = g_k$, then for all d sufficiently small, $g^*_a = G-2g_k$, and $g^*_j = g^*_k = g_k$.

PROOF OF CLAIM 2.4. By Claims 2.1 and 2.3, $g^*_j = g^*_k\geq g_k$. Suppose $g^*_j = g^*_k > g_k$ and consider $g''\in\Delta(G)$ such that $g'_j = g'_k = g^*_k-\varepsilon$ and $g'_a = g^*_k+2\varepsilon$ for some $\varepsilon\in\left(0,\min\left\{g^*_k-g_k,\frac{1}{2}(G-g^*_k)\right\}\right)$. Then $\widehat{U}_k(g'') = g''_k-\varepsilon > g^*_k = \widehat{U}_k(g)$, and $\widehat{U}_k(g'')-\widehat{U}_k(g^*) = 2\varepsilon$. This is a contradiction.

CLAIM 2.5. If $g_{\widehat{i}} = 0$ for some $\widehat{i}\neq a$, then $g^*_a = G$ and $g^*_i = 0$ for $i\neq a$.

PROOF of CLAIM 2.5. Since $\widehat{U}_{\widehat{i}}(g) = 0$, any policy can offer player $\widehat{i}$ at least his reservation value.

In the rest of the proof, denote the players without power by L and H, and assume that, $0 < g_L < g_H$. The maximization problem of an agenda setter can be divided into two stages. In the first stage, the agenda setter calculates his maximal utility if he forms a winning coalition with a certain player $i\in\{L,H\}$. Let $\widehat{g}^i$ be any policy that attains the maximum. In the second stage, the agenda setter compares $U_a\left(\widehat{g}^L\right)$ and $U_a\left(\widehat{g}^H\right)$ and chooses the optimal winning coalition.

CLAIM 2.6. For all d sufficiently small, $\widehat{g}^L_a = 1-2g_L$ and $\widehat{g}^L_L = \widehat{g}^L_H = g_L$.

PROOF OF CLAIM 2.6. By Claims 2.1 and 2.2, $\widehat{g}^L_L\geq\widehat{g}^L_H\geq g_L$. Suppose $\widehat{g}^L_L > g_L$ and consider $g''\in\Delta(G)$ such that $g''_L = \widehat{g}^L_L-\varepsilon$, $g''_H = \widehat{g}^L_H$ and $g''_a = \widehat{g}^L_a+\varepsilon$ for some

$\varepsilon \in \left(0, \min\left\{\widehat{g}_L^L - g_L, G - \widehat{g}_a^L\right\}\right)$. Then

$$\widehat{U}_L\left(g''\right) = (1-\beta)\left(g_L'' - \varepsilon\right) + \beta \min\left\{\widehat{g}_H^L, g_L'' - \varepsilon\right\} > g_L^* = \widehat{U}_L\left(g\right), \quad \text{and}$$
$$\widehat{U}_k\left(g''\right) - \widehat{U}_k\left(g^*\right) = (1-\beta)\varepsilon + \beta \max\left\{0, \widehat{g}_H^L - \left(\widehat{g}_L^L - \varepsilon\right)\right\} > 0,$$

which is a contradiction. Therefore, $\widehat{g}_L^L \geq \widehat{g}_H^L \geq g_L$.

CLAIM 2.7. For all d sufficiently small, $\widehat{g}_L^H > 0$.

PROOF OF CLAIM 2.7. Suppose $\widehat{g}_L^H = 0$. Then

$$\widehat{U}_H\left(\widehat{g}^H\right) - \widehat{U}_H\left(g\right) = (1-\beta)\left(\widehat{g}_H^H - g_H\right) + \beta\left(0 - g_L\right), \quad \text{and}$$
$$\lim_{d\to 0}\left[\widehat{U}_H\left(\widehat{g}^H\right) - \widehat{U}_H\left(g\right)\right] = -g_L < 0.$$

This is a contradiction.

CLAIM 2.8. For all d sufficiently small, $\widehat{g}_H^H + \widehat{g}_L^H = G$.

PROOF OF CLAIM 2.8. Suppose that $\widehat{g}_H^H + \widehat{g}_L^L < G$ and consider $g'' \in \Delta(G)$ such that $g_H'' = \widehat{g}_H^H + \varepsilon$ and $g_L'' = \widehat{g}_L^H - \left(\frac{1-\beta}{\beta}\right)\varepsilon$ for some $\varepsilon \in \left(0, \min\left\{G - \widehat{g}_H^H, \left(\frac{\beta}{1-\beta}\right) g_L''\right\}\right)$.[5] Then

$$\widehat{U}_H\left(g''\right) = (1-\beta)\widehat{g}_H^H + \beta\widehat{g}_L^H = \widehat{U}_H\left(\widehat{g}^H\right) \geq \widehat{U}_H\left(g, a\right), \quad \text{and}$$
$$\widehat{U}_a\left(g''\right) - \widehat{U}_a\left(\widehat{g}^H\right) = \left(2 + \tfrac{1}{\beta}\right)(1-\beta)\varepsilon > 0.$$

This is a contradiction.[6]

CLAIM 2.9. For all d sufficiently small, $\widehat{g}_a^H = 0$, $\widehat{g}_H^H = 1 - \widehat{g}_L^H$, and $\widehat{g}_H^H = \left(\frac{1}{2\beta-1}\right)\left[\beta\left(G - g_L\right) - (1-\beta) g_H\right]$.

PROOF OF CLAIM 2.9. Suppose d is sufficiently small. Since $\widehat{g}_H^H > 0$ by Claims 2.2 and 2.7, the constraint $\widehat{U}_H\left(\widehat{g}^H, a\right) \geq \widehat{U}_H\left(g, a\right)$ must be binding. Otherwise the agenda setter can reduce $\widehat{g}_H^H$ by $\varepsilon > 0$ sufficiently small and increase $\widehat{g}_a^H$ by ε, so that he increases his own utility while still satisfies player H by his reservation value. A binding constraint requires that

$$(1-\beta)\left(\widehat{g}_H^H - g_H\right) + \beta\left(\widehat{g}_L^H - g_L\right) = 0.$$

Together with Claim 2.8, there are two equations with two unknowns $\left(\widehat{g}_H^H, \widehat{g}_L^H\right)$. The proof is completed by solving this equation system. Note that for all d sufficiently small, $(1-\beta) g_H + \beta g_L \leq \frac{1}{2}G$, and therefore $\widehat{g}_L^H \leq \widehat{g}_H^H$, which is consistent with Claim 2.2.[7]

CLAIM 2.10. For all d sufficiently small,

$$\widehat{U}_a\left(\widehat{g}^L\right) = G - 2g_L, \quad \text{and}$$
$$\widehat{U}_a\left(\widehat{g}^H\right) = \beta\left[G - 2\left(\tfrac{1}{2\beta-1}\right)\left[\beta g_L - (1-\beta)\left(G - g_H\right)\right]\right].$$

[5] By Claim 2.7, $\widehat{g}_L^H > 0$ for all d sufficiently small, and therefore a legitimate ε is feasible.

[6] In the calculation of $\widehat{U}_H\left(\mathbf{g}'', a\right)$, $\min\left\{\widehat{g}_L^H, \widehat{g}_H^H\right\} = \widehat{g}_L^H$ by Claim 2.2.

[7] If $g_H = g_L$, then $(1-\beta) g_H + \beta g_L = g_H \leq \frac{1}{2}G$. If $g_H > g_L$, then $\lim_{d\to 0}(1-\beta) g_H + \beta g_L = g_L < \frac{1}{2}G$.

PROOF OF CLAIM 2.10. This directly results from Claims 2.6 and 2.9.

CLAIM 2.11. For all d sufficiently small, $\widehat{U}_a\left(\widehat{g}^L\right) > \widehat{U}_a\left(\widehat{g}^H\right)$ and therefore $g^* = \widehat{g}^L$.

PROOF OF CLAIM 2.11. By Claim 2.10,

$$\widehat{U}_a\left(\widehat{g}^L\right) - \widehat{U}_a\left(\widehat{g}^H\right) = \left(\tfrac{1-\beta}{2\beta-1}\right)\left(G - 2\beta g_H - 2\left(1-\beta\right) g_L\right) \quad \text{and}$$
$$\lim_{d\to 0} \widehat{U}_a\left(\widehat{g}^L\right) - \widehat{U}_a\left(\widehat{g}^H\right) = G - 2g_H > 0.$$

To conclude, the policy rule specified in the proposition is verified by Claims 2.4, 2.5, and 2.11.

■

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