Policy Dynamics in a Parliamentary Democracy with Proportional Representation

David P. Baron* Daniel Diermeier† Pohan Fong‡

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Abstract

This paper presents a dynamic model of government formation and policy choice in a parliamentary democracy with proportional representation in which the policy chosen in one period becomes the status quo for the next period. The electorate votes strategically by taking into account the likely governments that parties would form and the policies they would choose as a function of the status quo. The status quo also determines the bargaining power of the parties in government formation and their policy choice. A formateur thus has incentives to position strategically the current period policy to gain an advantage both in the next election and in the subsequent government formation. These incentives can result in policies that are outside the Pareto set of the parties and transitions of governments and policies.

*Graduate School of Business, Stanford University

†Kellogg Graduate School of Management, Northwestern University

‡Department of Economics, Concordia University

1 Introduction

How political constitutions influence policy choice is of central importance in the field of political economy. In recent years considerable progress has been made by developing comparative models of political institutions to predict the induced policy choices (e.g. Persson and Tabellini 2000, 2003). While these models have been successful in comparing certain dimensions of political institutions, e.g., by comparing parliamentary and presidential systems, other constitutional differences have received little attention. For example, there is only a limited understanding of whether there is an institutional explanation for the significant variation in economic policies and performance across parliamentary democracies. One reason for these gaps is the absence of general models of policy choice in parliamentary democracies, especially for the modal case of multi-party systems under proportional representation. Policy choice in a parliamentary democracy is affected by each of the three principal institutions–elections, government formation, and parliamentary authority. The perspective taken here is one of bargaining among parties over government formation and policy choice with representation determined by proportional representation. The paper provides a dynamic theory of policy choice, representation, and government that shows how policy in one period is affected by the incentives arising from the three institutions of parliamentary government in future periods. The result can be Pareto inefficiency.

The institutions of multi-party parliamentary democracies create incentives for both parties and voters to act strategically. Such incentives are well-understood in the case of plurality rule elections, but they are also present under proportional representation. First, incentives for strategic voting are created by minimum representation thresholds (Cox 1997, Austen-Smith and Banks 1988, Baron and Diermeier 2001). Supporters of parties with a small effective vote share may abandon their most preferred party and vote for a party with a larger expected vote share to avoid wasting their vote. These incentives are similar to those present in plurality systems that lead to phenomena such as Duverger's Law.[1] Additional incentives for strategic behavior are due to proportional representation, where it is rare for one party to capture a majority of seats.[2] In the typical case where no party

[1] See Cox (1997) for a detailed overview.

[2] The empirical regularity that under proportional representation there seldom is a majority party needs to be explained. See Baron and Diermeier (2001) for an explanation in the context of a one-period model. This issue is explored in more detail here.

commands a majority of seats, parties need to form governing coalitions by bargaining over officeholding benefits as well as policy. Because government policies are the consequence of multi-party coalition bargaining, voters may base their vote on the anticipated government policy, not on a party's announced platform or policy preferences. A moderate supporter of a conservative party, for example, may prefer a coalition government of the conservatives with a centrist party over a single-party conservative government. So, in cases in which the conservative party is close to gaining an absolute majority of seats, the voter may be (weakly) better off voting for the centrist party instead. This paper identifies the policy consequences of such sophisticated voting compared to sincere voting for the closest party.

A small but growing literature has considered these incentives in full-equilibrium models of parliamentary democracies (Austen-Smith and Banks, Baron and Diermeier). In these models agents act strategically, and the outcome constitutes a subgame perfect Nash equilibrium in government formation, policy choice, and elections. This literature, however, has not considered a potentially important incentive for strategic behavior.[3] Incumbent governments may strategically position the current government policy to influence the outcome of the next election, as well as the subsequent government formation and policy choice. This requires a dynamic analysis, where dynamics means that the policy chosen in one period become the status quo for the next period. Any new policy must defeat that status quo. This means that the incumbent government can choose a policy in the current period that both responds to its policy preferences and positions the status quo to advantage itself in the next election and government formation cycle.

Few political economy models have addressed the dynamics of policy choice, and none has incorporated elections. Baron (1996) considers a dynamic model with a unidimensional policy space and without elections and shows that the policy choices converge to the ideal policy of the median legislator. Baron and Herron (2003) consider a two-dimensional policy space with no elections, and both analytically and computationally study the government coalitions that form and their policies in a model with a finite horizon. Discontinuities in the value functions, however, precluded a general characterization of equilibria. In con-

[3]See, however, Fong (2006) who uses the Baron-Diermeier model to analyze the incentives of parties to strategically position the current policy to influence the bargaining over government formation and legislation in the next period. In contrast to Baron and Diermeier, however, Fong assumes sincere voting. This model is discussed in more detail below.

trast to these models, the present paper incorporates transferable benefits as well as policy preferences, which allows explicit characterization of equilibria.

Kalandrakis (2004) characterizes a Markov perfect equilibrium for a repeated divide-the-dollar game in which the status quo for one round is the allocation of the dollar in the previous round. He shows that the equilibrium transitions to allocations in which the proposer in each period takes all of the dollar. In his model preferences are linear, however, so there is no efficiency incentive to attain an allocation that responds to the policy preferences of coalition members. Bernheim, Rangel, and Rayo (2006) consider a more general, finite-horizon model in which the status quo in a round is the winner from the previous round. They show that the last proposer essential obtains its ideal outcome, and hence the outcome is not efficient unless the model is pure distribution. In the model presented in this paper the parties choose a policy that must be voted against the status quo, which is the policy in place in the previous period, but policy preferences are not linear and elections provide additional incentives for strategic behavior.

Battaglini and Coate (2005) characterize Markov perfect equilibria for dynamic models in which a legislature chooses the stock of a public good financed by distortionary taxes. In each period the legislature can increase or decrease the stock of the public good, so the legislature votes between the change in the stock and no change. In a 2006 paper they consider a legislature that can spend on both pork and a public good and can finance the spending with distortionary taxes and debt. Because of shocks to the value of the public good, inefficiency can result in too little spending on the public good and taxes and debt are too high. The political system considered is a legislature in office, so elections are not considered.

Riboni (2005) characterizes a Markov perfect equilibrium for a model in which the current status quo is the state variable, and any new policy must defeat that status quo. In the model presented here, a proposed policy must defeat the status quo, but in contrast to Riboni, who assumes that the agenda setter is fixed over time, the identity of the agenda setter is determined endogenously as a result of an election and the selection of a formateur. Riboni shows that the heterogeneity of preferences of voters on a committee can give rise to effective commitment for monetary policy. Penn (2005) takes a different approach to studying dynamics by considering the long-run preferences of players in a collective choice setting. In her model a proposal in the current period is voted against the status quo, which is the outcome of the vote in the previous period. In contrast to the model here, proposals in her model are generated exogenously rather than by the players. Her focus is on generating the value functions for players as the game continues indefinitely. She presents a computed spatial example in which policy outcomes are all in the Pareto set and are close to the stable set. In the model presented here the formateur can have incentives to propose policies outside the Pareto set to advantage itself in the subsequent election and government formation process.

Besley and Coate (1998) consider a two-period citizen-candidate model in which the citizen elected in first period can invest in a public good that has a cost in the first period and a return in the second period. In the equilibrium the identity of the citizen who will be elected in the second period may not be known with certainty because of a tie. In that case the first-period elected official may not undertake a Pareto-improving investment in a public good for fear that in the second period a citizen with opposing preferences will be elected and not compensate those who bore the cost of the public good in the first period. They view this as a political failure rather than a failure due to a commitment problem associated with costs and returns occuring at different points in time. In the model considered here, costs and benefits are contemporaneous, and any inefficiency is due to the incentives provided by the institutions of the parliamentary system.

To analyze the dynamics of representation, government formation, and policy choice, a multi-period equilibrium model of parliamentary democracies is developed based on Baron and Diermeier. A period consists of an election, a government formation stage, and the choice of a policy by parliament. A key feature of that model is that the equilibrium policies in a period are completely determined by that period's initial status quo policy. This induces a dynamic process of policy change where the current government's policy determines the status quo for the next period. Parties recognize that the current policy choice has consequences for the next period, so the governing parties have an incentive to choose the current policy strategically to create an advantage in the next period's election and government formation process. If the parties are sufficiently impatient, the optimal policy choice for the formateur is *not* to position the policy so that the formateur's party would receive a majority in the next period. Rather, it is optimal for the formateur to position the status quo to balance its current period policy preferences with the anticipated electoral outcome and its bargaining power in the next coalition government. The model thus can account for the fact that parliamentary democracies under proportional representation rarely yield a parliament with a majority party, even if the current government can strategically position the status quo for future elections. If the parties are sufficiently patient, however, the formateur positions the current period policy so that it receives a majority in the next election. Thus, the theory is consistent with the absence of majority parties when parties are impatient.[4]

Parties that take into account future political choices in addition to the present ones have centrifugal incentives that result in more extreme policies than would be chosen by myopic parties. When the future is sufficiently important, those policies are outside the single-period Pareto set of policy preferences. These policies are chosen not only to favor the formateur in the current period but also to disadvantage rival parties in the next election and in subsequent government formation. How extreme the policies are depends on how important the future is relative to the present. The more patient are parties the more extreme are the policies chosen in the first period. These extreme policies can lead to consensus government and centrist policies in the next period. Indeed, a centrist policy can be chosen by a majority party, but for a different reason. A majority of voters prefer to vote for a party that, if selected as the formateur, would form a consensus when the other parties if selected as formateur would form majoritarian governments with less central policies. Policies outside the Pareto set are only identified with majoritarian and single-party governments.

This paper thus identifies a strong form of inefficiency in policy-making that stems from the interaction of two institutions–elections and government formation. Parties have electoral incentives to obtain greater representation in parliament, since the likelihood of being selected as the formateur is weakly increasing in representation. Parties also have government formation incentives to position themselves favorably for the bargaining over policy and office-holding benefits in the next period. The instrument available to parties to respond to these incentives is the policy chosen in the current period.

[4]Alternative explanations of the absence of majority parties are considered in the final section. See Diermeier and Merlo (2003) for empirical support for the implication that parties are impatient.

2 The Model

The political system consists of large but finite (and even) number N of voters, and three political parties labeled a, b, and c. The political system selects a two-dimensional policy $x \in \Re^2$ in each of two periods. The choice in a period is made by a government formed among those parties that have representation in parliament as determined by a proportional representation electoral system. A government consists of a coalition of parties with a majority of seats in parliament. A two-period model is sufficient to identify the incentives for strategic behavior on the part of the political parties. The analysis begins after the first-period election and the selection of the formateur, since the policy choice in period one depends only on the coalition and the formateur.

In a period t party $i \in \{a, b, c\}$ derives utility from both policy x_t and the redistribution of office-holding benefits $y_t^i \in \Re$, where $y_t^a + y_t^b + y_t^c = 0$ and $y_t^i = 0$ if party i is not in government. The office-holding benefits can take a variety of forms. Baron and Diermeier (p. 935) give examples: "These benefits include jobs for party stalwarts, board seats on public companies or the national television system, and transfers to interest groups and party foundations. Again consider Germany. All the major parties (as well as interest groups like churches and labor unions) occupy seats on the supervisory boards of the national television system and major corporations (such as Volkswagen). Moreover, each major party receives substantial amounts of public money for its research and education foundations. Similar arrangements are common in many other parliamentary democracies, especially Austria and Italy."

The expected discounted sum of utility of party i in period t is given by

$$E\left[\sum_{t=1}^{2} \beta^{t-1} \left(y_t^i + u^i\left(x_t\right)\right)\right]$$

where $\beta \in [0, 1]$ is a common discount factor and the expectation is over the selection of formateur in the second period and any mixing of strategies. The party is also assumed to have lexicographic preferences over its status as formateur and utilities derived from policies and office-holding benefits. In particular, the party desires a higher level of utility regardless of the chances that he will be recognized as formateur, but whenever the utility is the same for two distinct bundles of policies and benefits, it prefers the bundle in which it is more likely to be recognized as formateur in the following period. This assumption is technical and helps eliminate additional equilibria in the first period. Substantively, the party's lexicographic preferences reflect a preference to head a government.

For the sake of tractability the policy preferences are assumed to be quadratic

$$u^i(x) = -\left\|x - z^i\right\|^2,$$

where $z^i \in \Re^2$ denotes party i's ideal point. Thus, parties not only prefer policies closer to their ideal points, but they are more averse to policy changes the farther those changes are from their ideal points.

So that there are no preference alignments among the parties, the ideal points of the three parties are assumed to form an equilateral triangle. This specification allows us to isolate the dynamics resulting from the policy choices to be isolated. Without loss of generality, we normalize the policy space so that $\left\|z^i - z^j\right\| = 1$ for all $i, j = a, b, c$ and $i \neq j$. The policy $\overline{z} \equiv \frac{1}{3}\sum_{i=1}^{3} z^i$ is the center of party preferences (or the centroid).

Voters care only about policy outcomes. The preferences of voter v in period t are represented by a time-separable utility function $u^v(x_t)$ of the same form as those of the parties. That is, the parties are formed among the electorate. The expected discounted utility of a voter is given by

$$E\left[\sum_{t=1}^{2} \delta^{t-1} u^v(x_t)\right],$$

where $\delta \in [0,1]$ is a discount factor that may differ from that of the political parties and their leaders. A voter v is characterized by his ideal point $z^v \in \Re^2$. The ideal points of voters are assumed to be uniformly distributed on $\mathcal{Z} \equiv \left\{x \in \Re^2 : \|x - \overline{z}\| < L\right\}$, where $L \geq \frac{1}{\sqrt{3}}$, so voter preferences do not favor a particular party or coalition.

Timing

A period is the length of an inter-election period and consists of three stages. The first stage is a parliamentary election that determines the seat shares of the parties in the parliament. The second stage is government formation, and the third stage is legislative and involves the choice of a policy by the parliament. Let q_{t-1} denote the status quo at the beginning of period t, where q_0 is the initial status quo.

Parliamentary Election Stage The electoral system is proportional representation with a minimum vote share m required for representation, where $m \in \left(0, \frac{1}{4}\right)$. The restriction $m < \frac{1}{4}$ allows for the possibility that all three parties are represented in a parliament with a majority party. If the vote shares ρ_t^i of all parties are at least m, their seat shares s_t^i are $s_t^i = \rho_t^i$. If only party j's vote share is less than m, it is not represented in parliament and the other parties have seat shares $s_t^i = \frac{\rho_t^i}{1-\rho_t^j}$, $i \neq j$. If two parties have vote shares less than m, the other party has a seat share of 1.

Government Formation Stage After an election one party is selected as the formateur. Selection is governed by a proportionality rule with the probability of selection equal to the party's seat share in parliament, unless one party has a majority of seats in which case it is selected as the formateur. The formateur in period t forms a government, which is a coalition C_t, i.e., a non-empty subset of the parties represented in parliament such that $\sum_{i \in C_t} s_t^i > \frac{1}{2}$.

Legislative Bargaining Stage In forming a government, the formateur makes a take-it-or-leave-it offer to the other members of the coalition. The offer specifies a policy proposal, $x_t \in \Re^2$, and an allocation of office-holding benefits; i.e., who should pay whom and by how much.[5] The formateur has an incentive to make an offer the coalition prefers to the status quo, so the government passes the new policy x_t, and the office-holding benefits are allocated as proposed.[6] A new period $t+1$ then begins with the status quo $q_t = x_t$. If any party in government rejects the offer, the status quo q_{t-1} is the policy in period t, and no redistribution of the office-holding benefits is made. The status quo for period $t+1$ then is $q_t = q_{t-1}$.

In both the stages of government formation and legislative bargaining, if the formateur is indifferent between two different choices, it is assumed to break the tie by forming a government or proposing a policy that leads to a higher aggregate utility of all voters. This technical assumption only plays a role in the second period, and it guarantees the existence of an optimal policy choice in the first period.

Terminology A parliament in which no party has a majority is referred to as a minority parliament and necessarily has three parties represented, whereas in a majority parliament one party with a majority of the seats. A consensus government includes all three parties, a majoritarian government is composed of two parties, and a single-party government is composed of a majority party.

[5] The government may be understood as being of cabinet form in which all government parties must agree on the policy choice.

[6] An offer to form a consensus government is conditional on both coalition partners accepting the offer. If either rejects the offer, the status quo remains.

3 Results

3.1 Electoral and Legislative Equilibrium in the Second Period

The lemmata below summarize results in Baron and Diermeier that characterize the equilibria for the second period. To facilitate our statement, we define $D^i \equiv \left\{ x \in \Re^2 : u^i(\mathbf{x}) > -\frac{1}{2} \right\}$, for all $i = a, b, c$, as the set of alternatives that yield party i a period payoff greater than $-\frac{1}{2}$. If the status quo is in D^i, party j as formateur prefers to form a consensus government rather than a majoritarian government with party k.

Lemma 1 ***Legislative equilibrium for the second period***

Consider the legislative bargaining stage in the second period with a status quo q_1. For a minority parliament: I:(A) A consensus government chooses the center of preferences $\overline{z}$. (B) A majoritarian government chooses the mid-point $z^{ij} = \frac{1}{2}\left(z^i + z^j\right)$, of the contract curve of the parties' ideal points for all $i, j = a, b, c$, $i \neq j$. (C) A single-party government chooses the ideal point of its member party $\mathbf{z}^i$, for all $i = a, b, c$.[7] *II: For all $i, j, k \in \{a, b, c\}$, $i \neq j \neq k$, party i as formateur (1) forms a consensus government if $q_1 \in \Re^2 \setminus \left(D^j \cup D^k\right)$, (2) forms a majoritarian government with party j if $q_1 \in D^j \cup D^k$ and $u^j(q_1) < u^k(q_1)$, and (3) forms a majoritarian government with party j or party k with probability $\frac{1}{2}$ if $q_1 \in D^j \cup D^k$ and $u^j(q_1) = u^k(q_1)$. III: The joint utility of all three parties is -1 when a consensus government is formed, $-\frac{5}{4}$ when a majoritarian government is formed, and -2 when a single-party government is formed. In a three-party parliament a majority party i chooses (A) a consensus government with policy $\bar{z}$ if $q_1 \notin D^j \cup D^k, j, k, \neq i$, (B) a majoritarian government with the party j that is the more disadvantaged by q_1 (for $q_1 \notin D^j$) and with policy at the midpoint z^{ij} of the contract curve if $q_1 \in D^j \cup D^k$ and $q \notin D^j \cap D^k, j, k \neq i$, and (C) a single-party government with policy z^i if $q_1 \in D^j \cap D^k, j, k \neq i$.*

Lemma 2 ***Electoral equilibrium for the second period***[8]

[7] A majority party, however, may not form a single-party government.

[8] This lemma restates Proposition 4 of Baron and Diermeier with one modification. The equilibrium vote shares in case (D-2) are different due to a different population structure. Baron and Diermeier assumed that voters' ideal points were uniformly distributed in the single-period Pareto set of the parties, whereas here voters' ideal points are assumed to be uniformly distributed in the disk $\mathcal{Z} \equiv \left\{ x \in \mathsf{R}^2 : \|\mathbf{z}^v - \overline{\mathbf{z}}\| < L \right\}$.

Consider the parliamentary election stage in the final (second) period with a status quo q_1. (A) If there is exactly one party i that as formateur would form a consensus government, every strong electoral equilibrium results in a majority parliament with three parties represented, where i is the majority party. The consensus government chooses policy $\overline{z}$. (B) If no party as formateur would form a consensus government, all strong electoral equilibria result in a minority parliament, a majoritarian government, and an even lottery over some pair $\{z^{ij}, z^{ik}\}$, $i \neq j \neq k$, of policies as the unique equilibrium policy outcome. (C) If all parties would form a consensus government, election of any three-party parliament is a strong electoral equilibrium and the policy is $\overline{z}$. (D-1) If party i as formateur would randomize between z^{ij} and z^{ik} and the other two parties would form governments with party i, a minority parliament results and the unique strong electoral equilibrium yields $\rho_2^i = m$, equal vote shares for the other two parties, majoritarian governments, and policy outcomes z^{ij} and z^{ik} with probability one-half. (D-2) If party i as formateur would form a majoritarian government and the other two parties j and k would form majoritarian governments with each other, a minority parliament results with a strong electoral equilibrium with expected vote shares $\rho_2^i = \frac{1}{2}$, $\rho_2^j + \rho_2^k = \frac{1}{2}$, and $\rho_2^j, \rho_2^k \in \left[m, \frac{1}{2}\right)$.

Lemma 2 I(A) identifies the intuition underlying both the bargaining within the coalition and the incentives to position the status quo strategically for the final period. A majority party in the last period could choose its ideal policy and form a single-party government. It prefers, however, to form a consensus government because bargaining with the other two parties generates the transfer of officeholding benefits from those parties in exchange for a policy closer to their ideal points. This bargaining is driven by convex preferences and continues until the policy is equi-distant from the three ideal policies; i.e., at $\overline{z}$. Indeed, the voters gave one party a majority because they anticipated that it would choose the centrist policy $\bar{z}$. In the previous period parties have an incentive to position the status quo so that they will have a majority in the final period and hence be selected as the formateur. Despite the coalition operating under unanimity, it is the formateur who controls the coalition bargaining by making a take-it-or-leave-it offer to the other parties. Hence, that party receives the officeholding benefits. Whether the government in the first period will position the status quo so that one party receives a majority depends on the impatience of the parties and is the subject of Section 3.2.2.

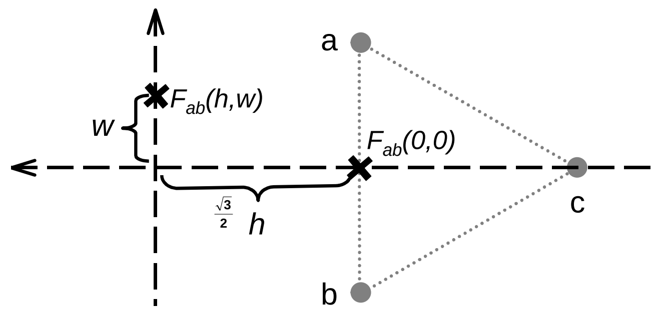

Figure 1: Transformation.

3.2 Legislative Equilibrium in the First Period

To characterize the legislative equilibrium in the first period, let $H_i(C)$ be the optimal policy choice by party i as formateur when it forms a government coalition C in the first period. Since parties care about their status as formateur (or leaders of government) in addition to the policy, a formateur may propose a policy that yields a greater chance of getting more votes in the subsequent parliamentary election.

Let $U^i(q_0)$ be the reservation value of party i in the first period given an initial status quo q_0. This is the expected discounted sum of utilities of party i if the initial status quo q_0 remains in effect in the first period due to a failure of government formation or coalition bargaining. Note that $U^i(q_0)$ is the sum of party i's period-one utility with the status quo policy q_0 and no side payments, plus its discounted continuation value for period 2 with a status quo $q_1 = q_0$. Given this notation, the expected discounted sum of utilities of party i in the first period is $y_1^i + U^i(x_1)$ if a policy x_1 is chosen and it receives office-holding benefits of y_1^i.

To facilitate the presentation of results, a notation system that locates the positions of different policies is used. For any $x \in \Re^2$ and for any distinct $i, j = a, b, c$, there exist $h_{ij}, w_{ij} \in \Re$ such that

$$x = F_{ij}(h_{ij}, w_{ij}) \equiv \frac{1}{2}\left(z^i + z^j\right) + \left(\frac{1}{2}\left(z^i + z^j\right) - z^k\right) h_{ij} + \left(z^i - z^j\right) w_{ij}.$$

(See Figure **??** for an illustration.) With the coordinate system of $F_{ij}(\cdot)$, any policy is described according to its position relative to the ideal points of all three parties. Note that if $h_{ij}{>}0$, the policy x is outside the Pareto set. For example, $F_{ab}(\frac{1}{\sqrt{3}}, 0) = (-\frac{1}{2}, 0)$.

3.2.1 Sincere voting in parliamentary elections

Fong (2006) analyzed a version of this game in which there is no parliamentary election and in each period the formateur is randomly selected from all three parties. Given the specification of voter preferences, his results correspond to those in a model with parliamentary elections in which each voter loyally casts her votes for the party whose ideal point is closest to hers.[9] In this case, we say those voters vote sincerely.

With sincere voting voters vote for the party closest to them and do so independently of the status quo, so parties cannot position the status quo to gain electoral advantage. Each party thus receives one-third of the vote. Despite a fixed electoral outcome, parties have an incentive to position the status quo strategically for the next period so as to obtain a bargaining advantage if selected as the formateur in the next period. The equilibrium policy choices by different governments are illustrated in Figure 2.

Proposition 1 *Suppose that in the second period all voters vote sincerely; i.e., every party gets votes from their natural constituencies and therefore has a vote share of one-third. In the first period: (A) A consensus government chooses the center of preferences $\overline{z}$, and a majoritarian government then is formed in period 2 and chooses the mid-point of the parties' contract curve as the policy. (B) Any majoritarian government $C = \{ij\}$ chooses a policy that is far away from the ideal point of the out party k. Moreover, this policy choice is outside the stage-game Pareto set of the three parties. In particular, there exists $\overline{\beta} \in (0,1)$ such that*

$$H_i(ij) = \begin{cases} F_{ij}\left(\frac{2\beta}{6-\beta}, 0\right) & \text{if } \beta \in \left[0, \overline{\beta}\right) \\ F_{ij}\left(\frac{1}{\sqrt{3}}, 0\right) & \text{if } \beta \in \left[\overline{\beta}, 1\right). \end{cases}$$

For all $\beta \in \left[0, \overline{\beta}\right)$, the policy choice is such that in the second period party i or j as formateur will form a majoritarian government with party k, whereas party k as formateur will randomize between majoritarian governments ik and jk. For all $\beta \in \left[\overline{\beta}, 1\right)$, the policy choice is such that in the second period a consensus government with policy $\bar{z}$ is formed with probability one.

Proposition 1 result identifies the formateur's incentive to position the status quo in the first period to gain a bargaining advantage. Suppose party a with $z^a = (0, \frac{1}{2})$ is the

[9] Baron (1993) makes this assumption in a model of endogenous party formation.

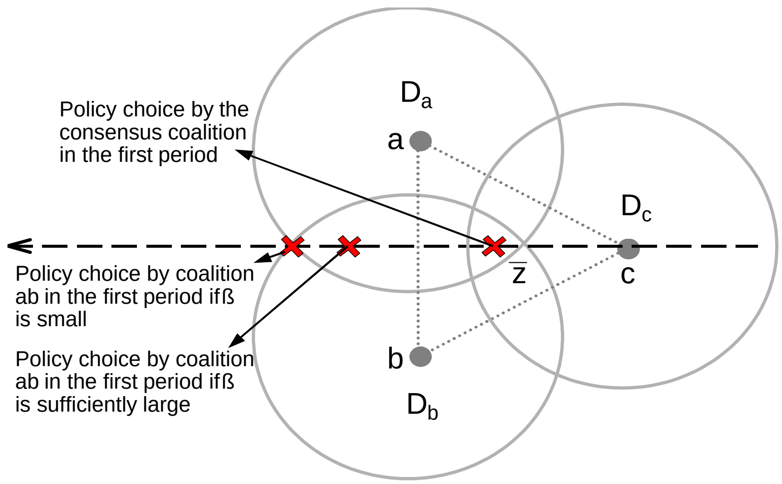

Figure 2: Equilibrium policy choices in the first period with sincere voting in parliamentary elections

formateur in the first period, and suppose that a prefers to form a majoritarian government with party b with $z^b = (0, -\frac{1}{2})$. In the absence of a second period ($\beta = 0$), party a chooses $x = z^{ab} = (0, 0)$, the midpoint of the contract curve of the two parties. As the future becomes more important (β increases), party a proposes a policy equidistant from z^a and z^b but farther from the ideal policy of the out party and thus outside the Pareto set. Doing so allows party a, if it is the formateur in period 2, to obtain greater officeholding benefits from party c in a majoritarian government, since the status quo disadvantages party c. Party b, if it is selected as the formateur in period 2, choose the same policy as party a would choose as formateur. Party b is advantaged by the status quo, but party a extracts the gain from b in the form of additional officeholding benefits, since the (expected) discounted gain to b in the second period is fully anticipated. If party c is selected as formateur, it forms a majoritarian government with either party a or b and is able to extract more officeholding benefits from its government partner than if the status quo were $(0, 0)$. In expectation party a as the period-one formateur is advantaged because of the convex preferences of the parties. As β increases above $\bar{\beta}$, the period-one formateur can extract no additional benefits because the centrist policy $\bar{z}$ is chosen in the second period.

3.2.2 Strategic voting in parliamentary elections

In this section the legislative equilibrium in the first period is characterized for the case in which voters are strategic in parliamentary elections. That is, voters anticipate not only the election outcome but also the possible outcomes of the government formation and policy choice stages. Each voter is pivotal for representation in parliament and hence for the probability that a party is selected as formateur in the second period.

To identify the incentives and resulting behavior of the parties and to trace the effects on policy, it is sufficient to begin the analysis after the election in the first period and after the selection of the first-period formateur. Equilibria are characterized for each of the types of government the formateur might form. Propositions 2, 3, and 4 characterize, respectively, the policy choice made by a consensus government, majoritarian government, and single-party government in the first period. Which type of government forms in the first period depends on the initial status quo q_0 and the identity of the formateur. The mapping from the initial status quo to the first-period election outcome, selection of formateur, and choice of government is both complex and discontinuous. A complete characterization of this mapping is very complex and is not informative about the dynamics of either governments or policy, so the choice of first-period government is not considered here. The following propositions are thus conditional on the type of government and the formateur selected. The proofs are provided in the Appendix.

Proposition 2 *(A) In the first period, a consensus government in the first period chooses policy* $\bar{z}$ *if the parties are sufficiently impatient; i.e.,* $H_i(abc) = \overline{\mathbf{z}}$ *for* $i = a, b, c$ *and* $\beta \in \left[0, \widehat{\beta}\right)$ *where* $\widehat{\beta} \equiv 4 - 2\sqrt{3} \simeq 0.536$*. In the second period there is a minority parliament, and each majoritarian government is formed with probability one-third. The policy for any chosen government* jk *is the midpoint* z^{jk} *of the contract curve. (B) If* $\beta \in [\hat{\beta}, 1)$ *and the first-period formateur* i *chooses to form a consensus government, the policy outcome is* $H_i(abc) = F_{jk}(-\frac{1}{\sqrt{3}}, 0)$, $j, k \neq i$*. In the second period party* i *receives a majority vote share and as formateur forms a consensus government with policy* $\overline{z}$*.*[10]

If parties are impatient ($\beta < \hat{\beta}$), a consensus government in the first period chooses the centroid $\bar{z}$, which is followed by a majoritarian government in the second period. With im-

[10] The policy $F_{jk}\left(-\frac{1}{\sqrt{3}}, 0\right)$ is equi-distant from the ideal points of parties j and k and relatively closer to that of formateur i.

patient parties the period-one formateur of a consensus government cannot gain sufficiently by strategically positioning the status quo for the second period. If parties are patient ($\beta \geq \hat{\beta}$), the future is sufficiently important that a first-period consensus government has an incentive to position the status quo to its advantage in the second period. It chooses a first-period policy close to its ideal point; e.g., if party c with ideal point $z^c = (\frac{\sqrt{3}}{2}, 0)$ is the formateur, the policy is $x_1 = (\frac{1}{2}, 0)$. This policy disadvantages parties a and b in the next election, so party c receives a majority. The policy x_1 also disadvantages parties a and b in the bargaining over government formation and policy choice in the second period, and party c makes policy concessions to them in exchange for officeholding benefits. Consensus governments always chooses a first-period policy that is interior to the single-period Pareto set, but that policy maximizes aggregate welfare in the first period only if the parties are impatient.

Proposition 3 *In the first period, any majoritarian government ij chooses a policy that is distant from the ideal point of the out party k. Moreover, the policy choice is outside the stage-game Pareto set of the three parties for all positive β. In particular, for any $m \in \left[0, \frac{1}{4}\right)$, there exists a decreasing function $\beta^* (m)$ such that*[11]

$$H_i (ij) = \begin{cases} F_{ij} \left(\frac{(1-m)\beta}{2-(1-2m)\beta}, 0 \right) & \text{if } \beta \in [0, \beta^* (m)) \\ F_{ij} \left(\kappa \left(\frac{\beta}{2-\beta} \right), |\kappa - 1| \frac{1}{2} \right) & \text{if } \beta \in [\beta^* (m), 1), \end{cases}$$

$$\text{where } \kappa \equiv \left(\frac{1}{2}\right)^{\frac{1}{2}} \left[\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2} \left(\frac{\beta}{2-\beta} \right) \right)^2 \right]^{-\frac{1}{2}}.$$

For all $\beta \in [0, \beta^* (m))$, the policy choice is such that in the second period both parties i and j obtain a vote share $\frac{1-m}{2}$ and party k obtains a vote share of m. In period two the formateur forms a majoritarian government and the policy outcomes are $H_i (ik)$ and $H_j (jk)$ with probability one-half each. For all $\beta \in [\beta^* (m), 1)$, the policy choice favors the period-one formateur. As a consequence, in the second period there is a majority parliament with all parties represented, where the period-one formateur receives a majority vote share and forms a consensus government with policy $\overline{z}$.

A majoritarian government always chooses the first-period policy strategically to position the status quo for the second period to favor the period-one formateur. When the

[11] The value of $\beta^* (m)$ is approximately 0.25 for all $m \in \left[0, \frac{1}{4}\right)$. The function $\beta^*(m)$ is characterized in the Appendix.

parties are very impatient, the policy is equi-distant from the ideal points of the government parties and both receive $\frac{1-m}{2}$ of the votes.[12] When the future is sufficiently ($\beta > \beta^*(m)$) important, the period-one formateur chooses a policy closer to its ideal point to disadvantage both of the other parties in the period-two election and bargaining. The first-period policy is then outside the Pareto set to provide the formateur with both an electoral and a bargaining advantage.

The next proposition characterizes how a single-party government of a majority party chooses a first-period policy.

Proposition 4 *In the first period, any single-party government i chooses a policy that favors itself but is far away and equally distant from the ideal points of the other parties. In particular, for all $\beta \in [0,1)$ and all distinct $j,k \neq i$,*

$$H_i(i) = \begin{cases} F_{jk}\left(-\frac{1}{1-2\beta}, 0\right) & \text{if } \beta \leq \beta^o \equiv \frac{1}{2}\left(\frac{\sqrt{2}}{\sqrt{2}+\sqrt{3}}\right) \\ F_{jk}\left(-1-\sqrt{\frac{2}{3}}, 0\right) & \text{if } \beta > \beta^o. \end{cases}$$

As a consequence, in the second period there is a majority parliament with all three parties represented, and the majority party i forms a consensus government with policy $\overline{z}$.

A majority party in the first period chooses a policy that yields it a majority in the period-two election. It receives a majority because it would as formateur choose the centrist policy $\bar{z}$, whereas the other two parties as formateur would form majoritarian governments with less central policies in period two. Hence, a majority of voters vote for the first-period majority party. The first-period policy is the formateur's ideal policy when $\beta = 0$, and for $\beta > 0$ it is outside the single-period Pareto set and equi-distant from the ideal policies of the other two parties. By choosing a policy away from the ideal points of the two opposing parties, the majority party can obtain more office-holding benefits in the second period by bargaining with both parties. For $\beta \geq \beta^o$ the policy $H_i(i) = F_{jk}\left(-1-\sqrt{\frac{2}{3}}, 0\right)$ is as far as possible from the ideal points of the opposing parties and still have party i receive a majority of the vote in period two. This creates the maximal bargaining power for party i in period two.

Legislative Bargaining

[12] Voters give the out party a vote share of m to keep it in parliament so that the other two parties will form governments with it.

To illustrate the bargaining, suppose that party a is the formateur in the first period. Party a knows that if it forms a government with policy x_1 in the first period, in the second period that policy will be the status quo q_1. The formateur in the second period will then form a government to maximize its utility given q_1. Suppose party a will also be the formateur in period 2 with probability 1. It could form a government with its ideal policy z^a, but it may prefer to make policy concessions to at least one of the other two parties, since those other parties are willing to compensate it by providing office-holding benefits.

Suppose that $q_1 \in D^a \setminus (D^b \cup D^c)$, so by Lemma 1 party a prefers to form a consensus government in period two with $x_2 = \bar{z}$ and parties b and c prefer to form majoritarian governments. Party $j = b, c$ will provide benefits $(-y^j)$ satisfying the coalition participation condition

$$u^j(\bar{z}) + y_2^j \;\geq\; u^j(q_1),$$

so

$$y_2^j = u^j(q_1) - u^j(\bar{z}).$$

Note that j's officeholding benefits are strictly lower as the status quo q_1 is farther from j's ideal point. That is, the formateur's bargaining power is greater the farther the status quo is from the ideal points of its possible government partners.

The utility or continuation value $v^a(q_1)$ of party a for a consensus government in period two is

$$\begin{aligned} v^a(q_1) &= u^a(\bar{z}) - y^b - y^c \\ &= u^a(\bar{z}) + u^b(\bar{z}) + u^c(\bar{z}) - u^b(q_1) - u^c(q_1), \end{aligned}$$

and the continuation values $v^j(q_1)$ are

$$v^j(q_1) = u^j(q_1), \;\; j = b, c.$$

The policy in period two maximizes the aggregate utility of the government parties and thus is efficient from the perspective of the parties; i.e., it is coalition efficient. When that policy is the centroid, it also maximizes the aggregate utility of the parties and also of the voters, since their ideal points are symmetrically located in a disk centered around the ideal points of the parties.

In the first period suppose that the formateur a were to form a majoritarian government with party b with policy $x_1 \in D^a \setminus (D^b \cup D^c)$. Party b prefers to be in government if

$$u^b(x_1) + y_1^b + \beta v^b(x_1) \geq u^b(q_0) + \beta v^b(q_0),$$

since the status quo for period 2 will be $q_1 = x_1$. Consequently, the government is formed with policy x_1 and a benefits transfer $y_1^b = u^b(q_0) + \beta v^b(q_0) - (1+\beta)u^b(x_1)$ from party b to party a.

The utility $W^a(x_1)$ of party a for the two periods then is

$$\begin{aligned}
W^a(x_1) &= u^a(x_1) - y_1^b + \beta v^a(x_1) \\
&= u^a(x_1) + (1+\beta)u^b(x_1) - u^b(q_0) - \beta v^b(q_0) + \beta(u^a(\bar{z}) + u^b(\bar{z}) + u^c(\bar{z}) - u^b(x_1) - u^c(x_1)) \\
&= u^a(x_1) + u^b(x_1) - u^b(q_0) - \beta v^b(q_0) + \beta(u^a(\bar{z}) + u^b(\bar{z}) + u^c(\bar{z}) - u^c(x_1)).
\end{aligned}$$

If it prefers to have a majority in period two, party a chooses the policy x_1^* satisfying

$$x_1^* \in \underset{x_1 \in D^a \setminus (D^b \cup D^c)}{\arg\max} W^a(x_1).$$

This policy is chosen to disadvantage party c both electorally and in the government formation in period 2, since in that period c is willing to provide sufficient officeholding benefits to obtain $\bar{z}$ rather than x_1. As in a single-period model, the period-one policy x_1^* maximizes the aggregate utility of the period-one government parties despite the formateur preferring to position the status quo for period two to advantage itself in the election and bargaining over government formation in period two. This results because the utility of the government partner is held to its reservation value.

In equilibrium the first-period formateur party a chooses x_1^*, party b joins the government, and party c does not. Given $x_1^* \in D^a \setminus (D^b \cup D^c)$, voters understand that if recognized as the formateur in period two, parties b and c would form majoritarian governments with policies on the contract curve of the parties in government. Voters also recognize that party a as the formateur in period 2 would choose a consensus government with the central policy $\bar{z}$. A majority of voters prefers the central policy to the policies the other parties would choose, and hence a majority of voters vote for a with the other voters vote for b and c. As the majority party, a is selected as the formateur in period 2, and bargaining with the other two parties results in the policy $\bar{z}$, as voters anticipated.

The period-one policy always favors the formateur, and possibly its government partner, and balances its utility in the first period with its utility in the second period. To illustrate this, consider a's policy choice as a function of β. For $\beta = 0$ the policy is $x_1^* = (0, 0)$, which is the midpoint of the contact curve of parties a and b. As β increases, the formateur proposes a policy equidistant from z^a and z^b but farther from z^c, and hence outside the single-period Pareto set. At $\beta = \sqrt{3} - 1$ it reaches $x_1^* = (-\frac{1}{2}, 0)$, which is the intersection of the boundaries of D^a and D^b. For $\beta \geq \beta^*(m) \approx 0.25$ the future is sufficiently important that the formateur forms a government with b at a policy sufficiently far from z^b and z^c that a will receive a majority in the next election. That is, as β increases, the policy moves along the boundary of D^a, but farther from both parties b and c. At $\beta = 1$ the policy is $x_1^* = (-\frac{\sqrt{3}}{2\sqrt{2}}, \frac{1}{2}(1 - \frac{1}{\sqrt{2}}))$. For all $\beta > \beta^*(m)$ party a as formateur in period 1 receives a majority of the vote and chooses a consensus government in period two.

This analysis identifies the importance of strategic voting. With sincere voting the vote shares (and seat shares) of the parties are each $\frac{1}{3}$, so no party can ever have a majority. With strategic voting voters choose the party to support based on the policies that will be chosen by the governments that could form in the second period. A party that will form a consensus government will receive a majority of the vote if the other two parties would form majoritarian governments and thus choose less central policies. In essence, a majority of voters reward the party that will choose a centrist policy. Similarly, for $\beta < \beta^*(m)$ a party that will form a majoritarian government chooses a policy that yields it a $\frac{1-m}{2}$ vote share. That is, voters reward it because it and its government partner will form period two governments with a more central distribution of policies than will the out party. With sincere voting voters cannot reward a party.

Political Failure

Proposition 5 identifies a political failure when the future is important (β sufficiently high). The term political failure is used here to refer to incentives inherent in the political system that lead to a policy that is outside the single-period Pareto set of party preferences.[13] In the theory presented here, political failures are associated with the institution of elections and with government formation and legislation in parliament. These failures are unavoidable, since voting is an inalienable right and voters and paries are unable to commit to future actions. The commitment problem in principle could be resolved by repetition, but the coordination problems among voters and among parties with divergent preferences

[13] Depending on how dispersed are voter preferences, the policy could also be outside the Pareto set of voter preferences. Voter preferences are considered further in Section 4.

seems insurmountable, particularly when political leaders are impatient.

The political failure associated with elections is the incentive of a formateur in the first period to position the status quo for the next period so that (1) it will have an incentive to choose a centrist policy in that period if selected as the formateur and (2) the other two parties will have an incentive to choose a less central policy. Then, the formateur in the current period is the only party that voters can count on to choose a centrist policy in the next period if chosen as the formateur. A majority of voters thus prefers to vote for the current formateur, and anticipating this, the formateur will position the policy, and hence the next status quo, outside the single-period Pateto set. This political failure is associated with majoritarian and single-party governments. The incentives provided by future elections are also present when parties are patient and a consensus government is formed in the first period. The resulting policy is interior to the Pareto set but not at the centroid.

One source of the political failure associated with elections is voters, who are willing to reward centrist policies in the final period even though it induces inefficiency in the previous period. This political failure results because voters cannot commit to how they will vote in future elections. If all voters were loyal to a party, and hence voted sincerely, commitment by voters would be assured. The presence of voters who respond to what the parties would do if in government, however, means that sincere voting would not be universal.

A second source of this political failure associated with elections is with the parties, which may have difficulty committing to enact, or not to enact, particular policies. A party's platform or a pre-announced electoral coalition could be credible, but only if voters vote sophisticatedly and would punish a party for deviating. Parties may be able to develop reputations for fulfilling promises, but political temptations to exploit a reputation for short-term gains can be substantial, particularly when voters are sophisticated and respond to their anticipation of subsequent behavior by the parties. Moreover, the Pareto inefficiency is greater when the future is important to the parties, which would make reputations difficult to sustain.

The political failure associated with government formation and legislative bargaining is evident from Proposition 1. Because of convex preferences a party has an incentive as formateur to position the status quo for the second period outside the single-period Pareto set. This increases its bargaining power in the second period. This failure is (weakly) more

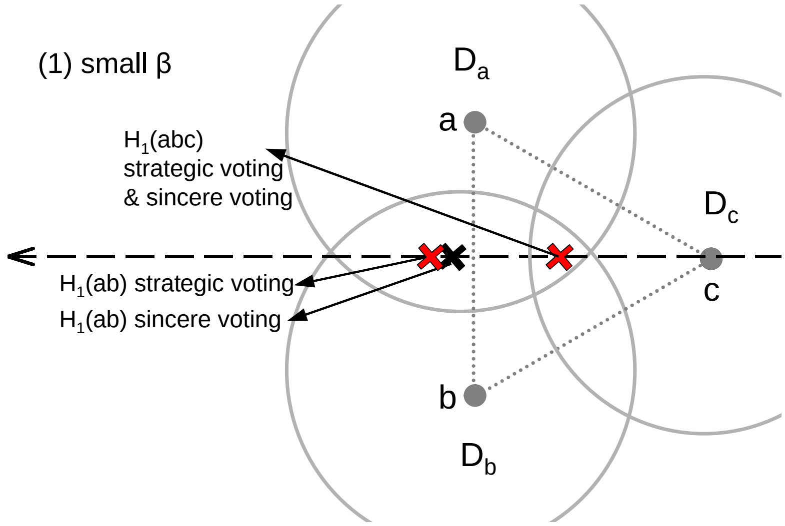

Figure 3: Equilibrium policy choices with $\beta \in (0, \beta^* (m))$.

severe the more patient are the parties, so reputation is unlikely to be a remedy. This bargaining failure is exacerbated by the political failure associated with elections which can lead to even more extreme policies as indicated in Proposition 3 for majoritarian governments and Proposition 4 for single-party governments. Again, this failure is more severe the more patient are the parties.

3.3 Dynamics of government coalition and policy choice

Propositions 2, 3, and 4 identify a rich set of dynamics of government coalition and policy choice. Given any representation hurdle m, three regions of political patience can be identified, each of which has a different pattern of dynamics.

1. $\beta \in (0, \beta^* (m))$: This case is illustrated in Figure 3. Properties: (i) A consensus government in the first period chooses the central policy $\bar{z}$, and neither the government nor the policy is sustainable in the second period. (ii) A majoritarian government in the first period chooses a policy that is outside the single-period Pareto set and equidistant from the ideal points of the government parties. This policy disadvantages the outparty in the next election and in government formation and treats the government parties identically. (iii) Conditional on either a consensus government or a majoritarian government in the first period, in the second period there is a minority parliament and a majoritarian government forms and chooses $z^{ij}, \forall i, j$; i.e., the mid-point of the

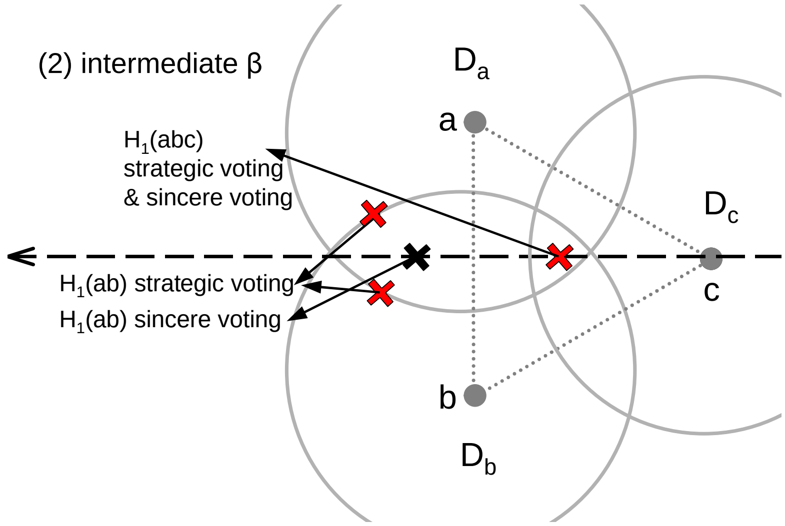

Figure 4: Equilibrium policy choices with intermediate $\beta \in \left[\beta^*(m), \widehat{\beta}\right)$.

contract curve of the government parties.

2. $\beta \in \left[\beta^*(m), \widehat{\beta}\right)$: This case is illustrated in Figure 4. Properties: (i) A consensus government in the first period chooses the central policy $\bar{z}$, and neither the government nor the policy is sustainable in the second period. (ii) There is a minority parliament in the second period only if a consensus government is formed in the first period. (iii) A formateur that forms a majoritarian government in the first period chooses a policy that gives it a majority in the election. In the second period the period-one formateur forms a consensus government with policy $\bar{z}$.

3. $\beta \in \left[\widehat{\beta}, 1\right)$: This case is illustrated in Figure 5 for $\beta \in \left[\widehat{\beta}, \widetilde{\beta}\right)$ and Figure 6 for $\beta \in \left[\widetilde{\beta}, 1\right)$, where $\widetilde{\beta} = \sqrt{3} - 1 \simeq 0.732$. Properties: (i) A formateur that forms a consensus government in the first period chooses a policy in the Pareto set that differs from the central policy and yields it a majority in the election. As a consequence, the consensus government is sustainable in the sense that it is formed again in the second period. The policy in period 2 is the centroid $\bar{z}$. (ii) A formateur that forms a majoritarian government in the first period chooses a policy outside the Pareto set that yields it a majority in the election. In the second period the period-one formateur forms a consensus government with policy $\bar{z}$. (iii) In the second period there is a majority parliament, regardless of the government formed and the policy in the first

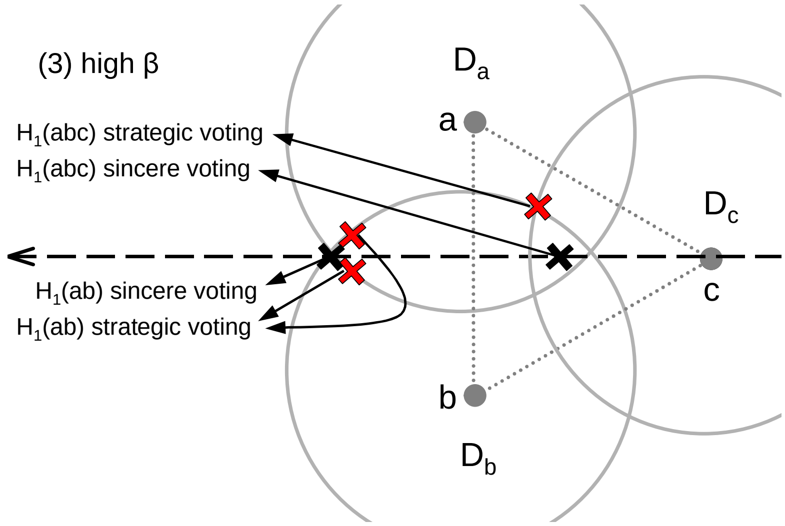

Figure 5: Equilibrium policy choices with high $\beta \in \left[\widehat{\beta}, \widetilde{\beta}\right)$.

period.

3.4 Comparative Statics–A Centrifugal Force

By comparing the policy outcomes in models with sincere and strategic voters, the effect of parliamentary elections on policy choice can be identified. In the first period, a consensus government may not be formed around a central policy, as it would be if the voters were to cast their votes sincerely. In particular, if the parties are sufficiently patient, i.e., β is sufficiently large, a formateur of a consensus government in the first period chooses a policy in the Pareto set that yields it a majority vote share in the second period. The same formateur then forms another consensus government but with a central policy. This is a consequence of the tradeoff between contemporaneous and future payoffs.

With strategic voters and β sufficiently high, a first-period formateur that forms a majoritarian government does not choose a policy that is equally distant from the ideal points of its member parties, as it would do with sincere voters. Instead, it chooses a policy outside the Pareto set that yields the first-period formateur a majority in the second period. This reduces the joint period-one utility of the two parties but guarantees that in the second period the first-period formateur obtains a majority in the election. A consensus government then is formed in the second period. Therefore, an asymmetric and Pareto inferior policy is associated with a majoritarian government if the parties sufficiently care about the future.

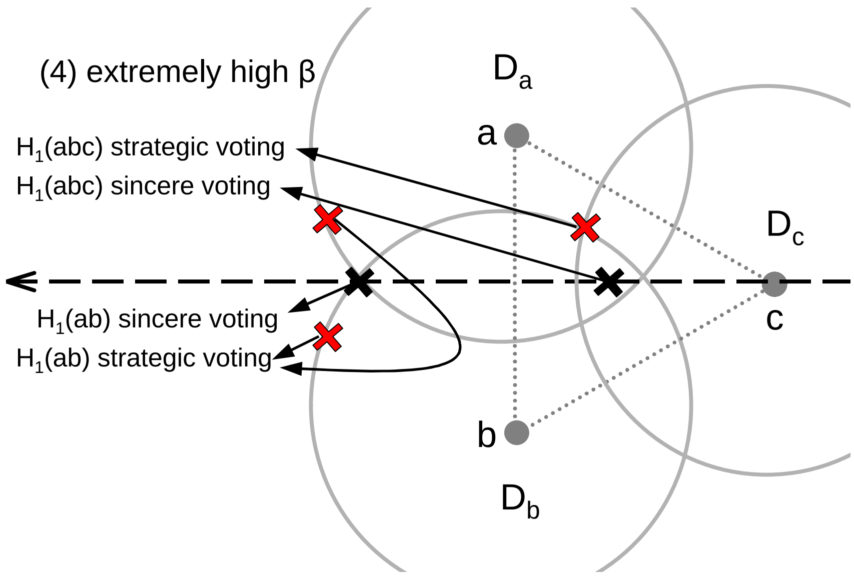

Figure 6: Equilibrium policy choices with high $\beta \in \left[\widetilde{\beta}, 1\right)$.

To measure the extremeness of a policy choice in the first period, define a metric, distance to the centroid (DTC). More specifically, for any policy $x \in \Re^2$, $DTC\,(x) \equiv \|x - \overline{z}\|$. Figure **??** shows the distance-to-centroid metrics of the policy choice of a majoritarian government in the first period. The solid line is associated with strategic voting, whereas the dashed line is associated with sincere voting. Except for a small range of intermediate values of β (associated with the outcomes illustrated in Figure 5), when voters vote strategically, in the first period a majoritarian government chooses a more extreme policy than it would have chosen had the voters voted sincerely. With both sincere and sophisticated voting a majoritarian government has an incentive to choose an extreme policy that is far from the out party's ideal point. This increases the bargaining power of the formateur by lowering the reservation value of the out party, allowing the two parties in the incumbent government to have a cheap coalition partner in the subsequent period. Moreover, the party strongly disadvantaged by the period-one policy choice is also handicapped in the following parliamentary elections because both parties in the incumbent government would form a majoritarian government with the out party from the first period. This induces the incumbent to choose an even more extreme policy in the first period. As a consequence, the probability that the out party is recognized as formateur in the second period is reduced further.

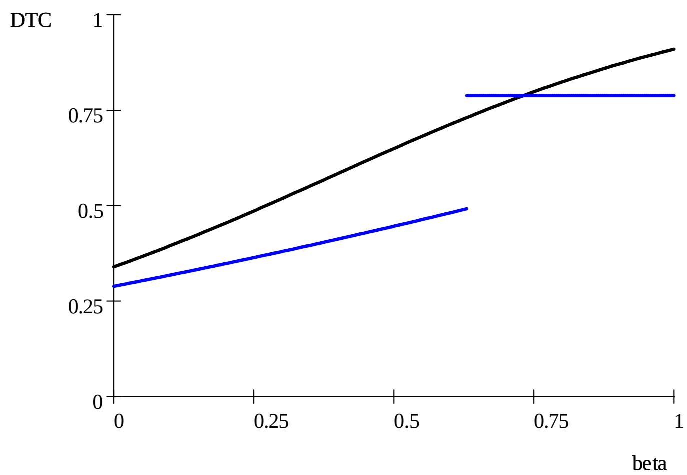

Figure 7: Distance to Centroid

Parliamentary elections with proportional representation thus provide a *centrifugal force* on policy choice in the first period in most situations, and that force is stronger with strategic voting than with sincere voting.

4 Welfare Analysis

This section examines how the institutions of parliamentary systems affect the welfare of voters and how social welfare responds to the impatience of political parties. Social welfare is defined as the average two-period utility of all voters. As shown in Appendix B, the average per-period utility is approximately a constant plus that of a hypothetical voter, whose ideal point is the central policy $\bar{z}$. Given any policy x, the aggregate per-period utility of all voters is then measured by $-\left\|x-\overline{z}\right\|^2$. Similarly, the aggregate two-period utility of all voters is measured by $-\left\|x_1-\overline{z}\right\|^2-\delta\left\|x_2-\overline{z}\right\|^2$.

As shown above, for most of the domain $[0,1]$ of β and conditional on any form of the government, the period-one policy outcome is more extreme if the voters vote strategically in the second period than if they vote sincerely. This implies that, conditional on any type of period-one government, on average voters are worse off if they vote strategically rather than sincerely. This results because sincere voting implicitly involves commitment to vote for the closest party regardless of the government that would be formed or the policy chosen in the second period, given the positioning of the status quo from the first period. Such sincere voting eliminates one source of political failure. Strategic voting reflects the inability of voters to commit to how they will vote, so their votes depend on the policy in the first period. The parties anticipate this and choose a policy that is more extreme. The inability of voters to commit thus gives rise to a moral hazard problem that reduces aggregate efficiency.

By Proposition 2 for all $\beta \in [0, \beta^* (m))$ or $\beta \in (\beta^* (m), 1)$, conditional on a majoritarian government being formed in the first period, the voters on average are worse of with a larger discount factor. This implies that social welfare is lower as the parties care more about their future utility. This results because the period-one formateur has an incentive to choose a more extreme policy the more important is the future. This achieves two goals for the formateur. First, it reduces the vote shares the other parties are likely to receive in the subsequent election. This increases the probability that the period-one formateur will again be recognized as the formateur in the second period. Second, once the period-one formateur is recognized as period-two formateur, it obtains greater office-holding benefits from its coalition partners, since they are more disadvantaged by the status quo and hence provide greater office-holding benefits. Note that when β moves from slightly below to slightly above $\beta^* (m)$, on average the voters are better off. This is because for $\beta \in (\beta^* (m), 1)$, the period-one policy is so extreme that in the second period a consensus government with a central policy $\overline{z}$ will result with probability one. This leads to a discrete jump of period-two utility for the average voter. The same result obtains if in the first period a concensus government is formed. That is, replace $\beta^* (m)$ in the above statement by $\widehat{\beta}$.

5 Conclusions

The principal institutions of parliamentary democracies are elections, government formation, and legislatures. Since the government serves with the confidence of the parliament, government formation and legislation are necessarily intertwined and a bargaining perspective is a natural approach to studying policy choice. Both government formation and legislation depend on representation in parliament, and the modal electoral institution is proportional representation. Political incentives arise from all three institutions, and both political parties and voters respond to the opportunities and incentives provided by those institutions. These incentives arise from both the current choices facing parties and voters and from future choices. The present and the future are linked by both long-lived players and the feature of political systems that the status quo policy remains in effect until it is replaced by a new policy. The policy chosen in the present period is the status quo for the next period, so the future shapes the incentives in the present period.

This paper identifies how the incentives present in a multi-party parliamentary system affect the dynamics of representation, governments, and policy across elections. The bargaining over government formation and policy choice creates intertemporal incentives, since the current policy choice affects the bargaining power of parties in the next period.[14] When parties are politically patient, bargaining incentives can lead to policies outside the single-period Pareto set, and the inefficiency is weakly increasing in political patience. Elections determine not only the representation of parties but also the likelihood that a party will be selected as formateur. This provides incentives for parties to position the current policy to provide an advantage in the next election. The incentives arise because voters anticipate both the governments that could form in the next period and the policies they would choose as a function of representation and the status quo. These electoral incentives can lead to policies farther from the center of voter preferences. Both bargaining and elections thus provide centrifugal forces on policy choice. These forces are generally stronger the more patient are political parties. These political failures are due to the institution of elections as well as the institutions of government formation and legislation.

The incentives present in a parliamentary system also affect the continuity of governments and policy. The incentives are sufficiently strong that governments generally do not persist from one period to the next and neither do their policies. Government transition and policy change thus should be expected in parliamentary systems. For example, inefficiency of the present policy can be followed by efficiency in the next period. The causation runs in the opposite direction, however. The incentives for a party to choose a centrist policy in the next period can lead a majority of voters to vote for that party, and in the present period that party has an incentive to position the status quo so that if it is the formateur in the next period it will choose a centrist policy. This advantages the party in both the election, because voters will reward it, and the subsequent bargaining over government formation.

The incentives leading to policy inefficiency and government and policy transition are due in part to commitment failures. If voters could commit to loyalty to a party, the cen-

[14] This insight is distinct from the literature on cabinet stability (e.g. Diermeier, Eraslan, and Merlo 2003, Warwick 1994) where the issue is the duration of governing coalition during an inter-election period.

trifugal force arising from elections would not be present. Similarly, if parties could commit to the governments they would form and the policies they would choose, the centrifugal force arising from bargaining over government formateur and policy would be mitigated but not eliminated.

The theory predicts that when parties are politically patient and voters vote strategically majority parties can arise. This prediction is contrary to empirical evidence: majority parties are rare in proportional representation systems. Several generalizations of the model could change this prediction. First, the model considers a parliamentary system with only three parties, and second, entry is not considered. With more parties or endogenous entry, majorities would be difficult to obtain. Third, if voters exhibit party loyalty; e.g., they vote sincerely, a majority party would not arise even with three parties.

The model considered here is sufficient to identify the incentives arising from the institutions of parliamentary systems and to provide predictions of the consequences of those incentives for government and policy continuity and for policy efficiency, but it is not fully dynamic. This is the subject of future research. As in any dynamic theory political patience of voters and parties is important, but the incentives for extreme policies identified here are stronger the more patient are players, which casts doubt on the role of norms or implicit agreements to overcome these inefficiencies.

Appendix

A Proofs of Propositions Pertaining to Legislative Equilibria in the First Period

Proof of Proposition 2. Consider a consensus government formed by party a in period 1. Note that a formateur will propose a policy that maximizes the joint two-period utility of all members in the government, since it can use redistributions of office-holding benefits as instruments to reallocate the utilities of the parties. Therefore, the proof involves the $H_a(abc)$ that maximizes the joint two-period utilities of all three parties in the first period.

Partition the policy space into two regions: $R_1^C \equiv \left(D^a \cap D^b\right) \cup \left(D^a \cap D^c\right) \cup \left(D^b \cap D^c\right)$ and $R_2^C \equiv \mathfrak{R}^2 \setminus R_1^C$. Note that neither R_1^C nor R_2^C is convex. In the second period, a status quo in R_1^C or R_2^C leads to a different joint payoff for all three parties. The approach is to characterize local maxima in these regions separately and then compare them to identify the globally optimal policy choice for party a. An optimal policy in a region R is denoted by $H_a(abc|R)$.

Region R_1^C Suppose that a consensus government is restricted to choose a policy from region R_1^C. Then by Lemmata 1 and 2, in the second period a majoritarian government will be formed, and the joint period-two payoff of all three parties will be $-\frac{5}{4}$. Therefore,

$$\sum_{i=a,b,c} U^i\left(H_a\left(abc|R_1^C\right)\right) = \max_{x' \in R_1^C} \sum_{i=a,b,c} u^i\left(x'\right) + \beta\left(-\tfrac{5}{4}\right) = -1 - \tfrac{5}{4}\beta,$$

and $H_a\left(abc|R_1^C\right) = \overline{z}$.

Region R_2^C Suppose that the consensus government is restricted to choose a policy in region R_2^C. Then by Lemmata 1 and 2, in the second period all three parties will be represented in parliament and a consensus government will be formed with policy $\overline{z}$. This implies that the joint period-2 payoff of all three parties will be -1. Therefore,

$$\sum_{i=a,b,c} U^i\left(H_1\left(abc|R_2^C\right)\right) = \max_{x' \in R_2^C} \sum_{i=a,b,c} u^i\left(x'\right) + \beta(-1) = \tfrac{\sqrt{3}}{2} - 2 - \beta,$$

and the maximum is attained at $F_{ab}\left(-\frac{1}{\sqrt{3}}, 0\right)$, $F_{ac}\left(-\frac{1}{\sqrt{3}}, 0\right)$, or $F_{bc}\left(-\frac{1}{\sqrt{3}}, 0\right)$. Note that if the last policy alternative is chosen, in the second period party a will receive a majority vote share and be recognized as formateur for certain. Therefore, $H_a\left(abc|R_2^C\right) = F_{bc}\left(-\frac{1}{\sqrt{3}}, 0\right)$.

Comparison Finally, it can be shown that $\sum_{i=a,b,c} U^i\left(H_1\left(abc|R_1^C\right)\right) > \sum_{i=a,b,c} U^i\left(H_1\left(abc|R_2^C\right)\right)$ if and only if $\beta \in \left[0, \widehat{\beta}\right)$ where $\widehat{\beta} \equiv 4 - 2\sqrt{3}$. ■

Proof of Proposition 3. Consider a majoritarian government ab, and determine the policy that maximizes the joint two-period utilities of parties a and b, i.e., $H_a(ab) = \max_{\mathbf{x}' \in \mathcal{R}^2} \sum_{i=a,b} U^i(\mathbf{x}')$.

Let $Q_{ij} \equiv \left\{x \in \Re^2 : u_i(x) > u_j(x) > u_k(x)\right\}$ for all $i, j, k = a, b, c$, $i \neq j \neq k$, and partition the policy space into 10 regions. In the second period, a status quo in a different region will lead to a different joint expected payoff of parties a and b. These regions are:

$R_1^T \equiv \left(\overline{D}^a \setminus \left(D^b \cup D^c\right)\right) \cup \left(\overline{D}^b \setminus \left(D^a \cup D^c\right)\right)$,

$R_2^T \equiv D^a \cap D^b \cap \{\mathbf{x} : u_a(\mathbf{x}) = u_b(\mathbf{x}) > u_c(\mathbf{x})\}$,

$R_3^T \equiv \Re^2 \setminus \left(\overline{D}^a \cup \overline{D}^b \cup \overline{D}^c\right)$,

$R_4^T \equiv D^a \cap D^b \cap (Q_{ab} \cup Q_{ba})$,

$R_5^T \equiv D^a \cap D^b \cap \left\{\mathbf{x} : \max_{i=a,b} \{u_i(\mathbf{x})\} > \min_{i=a,b} \{u_i(\mathbf{x})\} = u_c(\mathbf{x})\right\}$,

$R_6^T \equiv \left[(D^a \cap Q_{ac}) \cup \left(D^b \cap Q_{bc}\right)\right] \cap D^c$,

$R_7^T \equiv \left(D^a \cup D^b\right) \cap D^c \cap \left\{\mathbf{x} : \max_{i=a,b} \{u_i(\mathbf{x})\} = u_c(\mathbf{x}) > \min_{i=a,b} \{u_i(\mathbf{x})\}\right\}$,

$R_8^T \equiv \left(D^a \cup D^b\right) \cap D^c \cap \left\{x : u^a(x) = u^b(x) < u^c(x)\right\}$,

$R_9^T \equiv \{\overline{z}\}$, and

$R_{10}^T \equiv \overline{D}^c \setminus \left(D^a \cup D^b\right)$.

The following characterizes local maxima (or suprema if a maximum does not exist) region by region and then compares them to determine the (globally) optimal policy choice by government ab.

Region R_1^T Suppose that government ab is restricted to choose a policy from region R_1^T, and without loss of generality, suppose that a policy $x' \in \overline{D}_a \setminus \left(D^b \cup D^c\right)$ is chosen in the first period. Then by Lemma 2, in the second period all three parties will be represented, and party a will receive a majority vote share. By Lemma 1 party a as formateur will form a consensus government with policy $\overline{z}$, and the joint period-two payoff of parties a and b will be $(-1) - u^c(x)$. Therefore, the joint present value of parties a and b in the first period is

$$\sum_{i=a,b} U^i\left(x'\right) = \sum_{i=a,b} u_i\left(x'\right) + \beta\left(-1 - u_c\left(x'\right)\right), \tag{1}$$

and

$$\max_{x'\in\overline{D}^a\setminus\left(D^b\cup D^c\right)}\sum_{i=a,b}U^i\left(x'\right)=\begin{array}{ll}\max\limits_{x'} & -\left(2-\beta\right)\left\|x'-F_{ab}\left(\frac{\beta}{2-\beta},0\right)\right\|^2-\frac{(1+\beta)(1-\beta)}{2-\beta}\\ \text{s.t.} & \left\|x'-z^a\right\|^2\leq\frac{1}{2},\left\|x'-z^b\right\|^2\geq\frac{1}{2},\left\|x'-z^c\right\|^2\geq\frac{1}{2}.\end{array}$$

where the objective function on the right side is a simplification of equation (1), and the three constraint inequalities correspond to the constraints that $x'\in\overline{D}^a$, $x'\notin D^b$, and $x'\notin D^c$ respectively. The maximum is attained at

$$\begin{aligned}H_a\left(ab|\overline{D}^a\setminus\left(D^b\cup D^c\right)\right) &= F_{ab}\left(\kappa\left(\frac{\beta}{2-\beta}\right),|\kappa-1|\left(\frac{1}{2}\right)\right),\\ \text{where }\kappa &\equiv \left(\frac{1}{2}\right)^{\frac{1}{2}}\left[\left(\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\left(\frac{\beta}{2-\beta}\right)\right)^2\right]^{-\frac{1}{2}},\end{aligned}$$

and

$$\sum_{i=a,b}U^i\left(H_1\left(ab|\overline{D}^a\setminus\left(D^b\cup D^c\right)\right)\right)=-\left(\frac{2-\beta}{2}\right)\left(\frac{1}{\kappa}-1\right)^2-\frac{(1+\beta)(1-\beta)}{2-\beta}.$$

This is a corner solution, since either the constraint $\left\|x'-z^b\right\|^2\geq\frac{1}{2}$ or that $\left\|x'-z^a\right\|^2\leq\frac{1}{2}$ is binding.

Region R_2^T Suppose that a policy $x'\in R_2^T$ is chosen in the first period. Then by Lemma 2, in the second period party c will receive a vote share of m, and both parties a and b will receive $\frac{1-m}{2}$. As a consequence, with probability m party c will be recognized as formateur and randomize between majoritarian governments ac and bc. If, for example, a majoritarian government ac is formed, the joint period-two payoff of parties a and b will be $u^a\left(x'\right)+\left(-\frac{3}{4}\right)$, since party a is included in the new government and gets its reservation value and $\left(-\frac{3}{4}\right)$ is the period payoff to party b that is excluded from the government coalition. With probability $1-m$, either party a or b will be recognized, and the formateur will form a majoritarian coalition with party c. The joint period-2 payoff of parties a and b then will be $\left(-\frac{1}{2}-u^c\left(x'\right)\right)+\left(-\frac{3}{4}\right)$. Therefore, the joint discounted expected utility of parties a and b in the first period is

$$\sum_{i=a,b}U^i\left(x'\right)=\sum_{i=a,b}u^i\left(x'\right)+\beta\left(\left(1-m\right)\left(\left(-\frac{1}{2}-u^c\left(x'\right)\right)+\left(-\frac{3}{4}\right)\right)+m\left(u^a\left(x'\right)+\left(-\frac{3}{4}\right)\right)\right),$$

and

$$\sup_{x' \in R_2^T} \sum_{i=a,b} U_1^i(x') = \sup_{h \in \left(-\frac{1}{3}, \frac{1}{\sqrt{3}}\right)} \Phi(h) \equiv -\frac{3(2-(1-2m)\beta)}{4} \left(h - \frac{(1-m)\beta}{2-(1-2m)\beta}\right)^2 - \frac{4+2(1+4m)\beta-\left(5-8m-m^2\right)\beta^2}{4(2-(1-2m)\beta)}$$

$$= \begin{cases} -\frac{4+2(1+4m)\beta-\left(5-8m-m^2\right)\beta^2}{4(2-(1-2m)\beta)}, & \text{if } \beta \in \left[0, \widehat{\beta}(m)\right) \\ -1 + \left(\frac{\left(2\sqrt{3}-1\right)-2\left(2+\sqrt{3}\right)m}{4}\right)\beta & \text{otherwise.} \end{cases}$$

$$\text{where } \widehat{\beta}(m) = \begin{cases} 1 & \text{if } m \in \left[\widehat{m}, \frac{1}{4}\right), \ \widehat{m} \equiv 3\sqrt{3} - 5 \\ \frac{2}{1+\sqrt{3}-\left(2+\sqrt{3}\right)m} & \text{otherwise,} \end{cases}$$

For all $\beta \in \left[0, \widehat{\beta}(m)\right)$, the local supremum is attained by an interior policy $F_{ab}(h^*, 0) \in R_2^T$ where $h^* \equiv \frac{(1-m)\beta}{2-(1-2m)\beta}$, and therefore a supremum is a maximum. On the other hand, for all $\beta \in \left[\widehat{\beta}(m), 1\right)$, a maximum does not exist in region R_2^T. To see this, pick any policy $F_{ab}(h, 0) \in R_2^T$ and define $h(\varepsilon) \equiv \varepsilon h^* + (1-\varepsilon) h$. Since R_2^T is an open region, $F_{ab}(h(\varepsilon), 0) \in R_2^T$ for $\varepsilon > 0$ sufficiently small. Note that $\Phi(h)$ is strictly concave and $h^* = \arg\max_{h' \in \mathsf{R}} \Phi(h')$. Therefore, $\sum_{i=a,b} U^i(F_{ab}(h(\varepsilon), 0)) > \sum_{i=a,b} U^i(F_{ab}(h, 0))$. Finally, $\sup_{x' \in R_2^T} \sum_{i=a,b} U^i(x') \leq \max_{x' \in R_1^T} \sum_{i=a,b} U^i(x')$ for all $\beta \in [\beta^*(m), 1)$ and all $m \in \left[0, \frac{1}{4}\right)$, where $\beta^*(m)$ is a decreasing function in m and $\beta^*(m) \in \left(0, \widehat{\beta}(m)\right)$ for all m. This can be verified by comparing the functional forms of local maxima and/or suprema in regions R_1^T and R_2^T.

Region R_3^T Suppose that a policy $x' \in R_3^T$ is chosen in the first period. Then by Lemmata 1 and 2, in the second period all three parties will be represented, each party will be recognized as formateur with probability one-third (which is the probability they perceive before the period-two election), and a consensus government will be formed. Therefore, with probability one-third, party c will be recognized and the joint period-2 payoff of parties a and b will be $u^a(x') + u^b(x')$ because both of them will be included in the consensus coalition and receive their period-two reservation values. With probability two-thirds, either party a or b will be recognized, and their joint expected discounted utility will be $(-1) - u^c(x')$, which is the joint payoff of all three parties (that is, -1) net of party c's reservation value. Thus,

$$\sum_{i=a,b} U^i(x') = \sum_{i=a,b} u^i(x') + \frac{2}{3}\beta\left((-1) - u^c(x')\right) + \frac{1}{3}\beta \sum_{i=a,b} u^i(x'),$$

and

$$\begin{aligned}
\sup_{x' \in R_3^T} \sum_{i=a,b} U^i\left(x'\right) &= \sup_{x'} \; -2\left\|x' - F_{ab}\left(\tfrac{\beta}{3}, 0\right)\right\|^2 - \tfrac{1}{6}\left(1+\beta\right)\left(3-\beta\right) \\
&\quad \text{s.t.} \quad \left\|x' - z^a\right\|^2 > \tfrac{1}{2}, \left\|x' - z^b\right\|^2 > \tfrac{1}{2}, \left\|x' - z^c\right\|^2 > \tfrac{1}{2}. \\
&= -1 + \tfrac{1}{3}\left(\sqrt{3} - 1\right)\beta \\
&< \sum_{i=a,b} U^i\left(H_1\left(ab|R_1^T\right)\right)
\end{aligned}$$

for all m and β. Therefore, $H_a\left(ab\right) \notin R_3^T$.

Region R_4^T Suppose that government ab is restricted to choose a policy in region R_4^T, and without loss of generality suppose that a policy $x' \in D^a \cap D^b \cap Q_{ab}$ is chosen. By Lemma 2, in the second period all parties will be represented, and party a will receive a vote share of one-half. If party c is recognized as formateur, it will form a majoritarian government with party b, and the joint period-two payoff of parties a and b will be $u^b\left(x'\right) + \left(-\frac{3}{4}\right)$. It is perceived that this will happen with probability one-fourth (each legitimate electoral equilibrium is assumed to happen with equal "probability"). On the other hand, party a or b as formateur will form a majoritarian government with party c, and the joint discounted expected utility of parties a and b will be $\left(-\frac{1}{2} - u^c\left(x'\right)\right) + \left(-\frac{3}{4}\right)$. The probability of this event is three-fourths. Therefore, the joint discounted expected utility of parties a and b in the first period is

$$\sum_{i=a,b} U^i\left(x'\right) = \sum_{i=a,b} u^i\left(x'\right) + \beta\left[\tfrac{3}{4}\left(\left(-\tfrac{1}{2} - u^c\left(x'\right)\right) + \left(-\tfrac{3}{4}\right)\right) + \tfrac{1}{4}\left(u^b\left(x'\right) + \left(-\tfrac{3}{4}\right)\right)\right],$$

and

$$\begin{aligned}
\sup_{x' \in R_4^T} \sum_{i=a,b} U^i\left(x'\right) &= \sup_{F_{ab}(h,w) \in R_4^T} -\tfrac{3(4-\beta)}{8}\left(h - \tfrac{3\beta}{2(4-\beta)}\right)^2 - \tfrac{(4-\beta)}{2}w^2 - \tfrac{1}{4}\beta\left|w\right| - \left(\tfrac{64+64\beta-47\beta^2}{32(4-\beta)}\right) \\
&= -\left(\tfrac{64+64\beta-47\beta^2}{32(4-\beta)}\right) < \sup_{x' \in R_2^T} \sum_{i=a,b} U_1^i\left(x'\right),
\end{aligned}$$

for all m and $\beta \neq 0$. Therefore, $H_a\left(ab\right) \notin R_4^T$.

Region R_5^T Suppose that government ab is restricted to choose a policy in region R_5^T, and without loss of generality suppose that a policy $x' \in D^a \cap D^b \cap \left\{x : u^a\left(x\right) > u^b\left(x\right) = u^c\left(x\right)\right\}$ is chosen. By Lemma 2, in the second period all parties will be represented and party a will receive a vote share of one-half. If party a is recognized as formateur, it will randomize between majoritarian governments ab and ac, and the joint expected payoff of parties a and b in the second period will be $\frac{1}{2}\left(-\frac{1}{2}\right) + \frac{1}{2}\left(\left(-\frac{1}{2} - u^c\left(x\right)\right) + \left(-\frac{3}{4}\right)\right)$. The probability of this event is one-half. If party b is recognized as formateur, it will form a majoritarian government with party c, and the joint period-two payoff of parties a and b will be $\left(-\frac{1}{2}-u^{c}(x)\right)+\left(-\frac{3}{4}\right)$. Before the parliamentary election in the second period, it is perceived that the probability of this event is one-fourth. If party c is recognized as formateur, it will form a majoritarian government with party b, and the joint period-2 payoff of parties a and b will be $u^{b}(x')+\left(-\frac{3}{4}\right)$. Again, it is perceived that the probability of this event is one-fourth. Therefore, the joint discounted expected utility of parties a and b in the first period is

$$\begin{aligned}\sum_{i=a,b} U^{i}(x') &= \sum_{i=a,b} u^{i}(x')+\tfrac{1}{2}\beta\left(\tfrac{1}{2}\left(-\tfrac{1}{2}\right)+\tfrac{1}{2}\left(\left(-\tfrac{1}{2}-u^{c}(x')\right)+\left(-\tfrac{3}{4}\right)\right)\right)\\ &\quad+\tfrac{1}{4}\beta\left(\left(-\tfrac{1}{2}-u^{c}(x')\right)+\left(-\tfrac{3}{4}\right)\right)+\tfrac{1}{4}\beta\left(u^{b}(x')+\left(-\tfrac{3}{4}\right)\right),\end{aligned}$$

and

$$\begin{aligned}\max_{x'\in R_5^T}\sum_{i=a,b} U^{i}(x') &= \max_{F_{bc}(h,0)\in R_5^T} -\frac{3(8-\beta)}{16}\left(h+\frac{4}{8-\beta}\right)^2-\frac{40+48\beta-7\beta^2}{8(8-\beta)}\\ &= -\frac{40+48\beta-7\beta^2}{8(8-\beta)} < \sup_{x'\in R_2^T}\sum_{i=a,b} U^{i}(x'),\end{aligned}$$

for all m and $\beta\in[0,1)$. Therefore, $H_a(ab)\notin R_5^T$.

Region R_6^T Suppose that government ab is restricted to choose a policy in region R_6^T, and without loss of generality suppose that a policy $x'\in D^a\cap Q_{ac}\cap D^c$ is chosen. By Lemma 2, in the second period all parties will be represented, and party a will receive a vote share of one-half. If party a is recognized as formateur, it will form a majoritarian government with party b, and the joint period-two payoff of parties a and b will be $-\frac{1}{2}$. The probability of this event is one-half. If party b is recognized as formateur, it will form a majoritarian government with party c and the joint period-2 payoff of parties a and b will be $\left(-\frac{1}{2}-u^{c}(x')\right)+\left(-\frac{3}{4}\right)$. Before the parliamentary election in the second period, it is perceived that the probability of this event is one-fourth (by assumption that each legitimate electoral equilibrium happens with equal “probability"). If party c is recognized as formateur, it will form a majoritarian government with party b, and the joint period-two payoff of parties a and b will be $u^{b}(x')+\left(-\frac{3}{4}\right)$. Again It is perceived that the probability of this event is one-fourth. Therefore, the joint discounted expected utility of parties a and b in the first period is

$$\sum_{i=a,b} U^{i}(x')=\sum_{i=a,b} u^{i}(x')+\beta\left[\tfrac{1}{2}\left(-\tfrac{1}{2}\right)+\tfrac{1}{4}\left(\left(-\tfrac{1}{2}-u^{c}(x')\right)+\left(-\tfrac{3}{4}\right)\right)+\tfrac{1}{4}\left(u^{b}(x')+\left(-\tfrac{3}{4}\right)\right)\right],$$

and

$$\begin{aligned}\sup_{x'\in R_6^T}\sum_{i=a,b}U^i\left(x'\right) &= \sup_{F_{bc}(h,w)\in R_6^T} -\tfrac{3}{2}\left(h+\tfrac{1}{2}\right)^2-2\left(w-\tfrac{\beta+2}{8}\right)^2-\tfrac{16+20\beta-\beta^2}{32}\\ &= -\tfrac{5+6\beta}{8}<\sup_{x'\in R_2^T}\sum_{i=a,b}U^i\left(x'\right),\end{aligned}$$

for all m and β. Therefore, $H_a(ab)\notin R_6^T$.

Region R_7^T Suppose that government ab is restricted to choose a policy in region R_7^T, and without loss of generality suppose that a policy $x'\in D^a\cap D^c\cap\left\{x:u^a(x)=u^c(x)>u^b(x)\right\}$ is chosen. By Lemma 2, in the second period both parties a and c will receive a vote share of $\frac{1-m}{2}$ and party b will receive m. As a consequence, if party a is recognized as formateur, it will form a majoritarian government with party b, and the joint period-2 payoff of parties a and b will be $-\frac{1}{2}$. The probability of this event is $\frac{1-m}{2}$. If party b is recognized as formateur, it will randomize between majoritarian governments ab and bc, and the joint expected payoff of parties a and b in the second period will be $\frac{1}{2}\left(-\frac{1}{2}\right)+\frac{1}{2}\left(\left(-\frac{1}{2}-u^c(x)\right)+\left(-\frac{3}{4}\right)\right)$. The probability of this event is m. If party c is recognized as formateur, it will form a majoritarian government with party b, and the joint period-2 payoff of parties a and b will be $u^b(x')+\left(-\frac{3}{4}\right)$. The probability of this event is $\frac{1-m}{2}$. Therefore, the joint discounted expected utility of parties a and b in the first period is

$$\begin{aligned}\sum_{i=a,b}U_a^i\left(x'\right) &= \sum_{i=a,b}u^i\left(x'\right)+\beta\left(\tfrac{1-m}{2}\right)\left(-\tfrac{1}{2}\right)+\beta\left(\tfrac{1-m}{2}\right)\left(u^b\left(x'\right)+\left(-\tfrac{3}{4}\right)\right)\\ &\quad+\beta m\left(\tfrac{1}{2}\left(-\tfrac{1}{2}\right)+\tfrac{1}{2}\left(\left(-\tfrac{1}{2}-u^c(x)\right)+\left(-\tfrac{3}{4}\right)\right)\right),\end{aligned}$$

and

$$\begin{aligned}\sup_{x'\in R_7^T}\sum_{i=a,b}U^i\left(x'\right) &= \sup_{F_{ac}(h,0)\in R_7^T} -\tfrac{3(4+(1-2m)\beta)}{8}\left(h+\tfrac{2+(1-m)\beta}{4+(1-2m)\beta}\right)^2-\tfrac{20+(28-12m)\beta+\left(5-12m+m^2\right)}{8(4+(1-2m)\beta)}\\ &= -\tfrac{16+(19-2m)\beta}{24}<\sup_{x'\in R_2^T}\sum_{i=a,b}U^i\left(x'\right),\end{aligned}$$

for all m and β. Therefore, $H_a(ab)\notin R_7^T$.

Region R_8^T Suppose that a policy $x'\in R_8^T$ is chosen. By Lemma 2, in the second period all three parties will be represented and party c will get a vote share of one-half. If party c is recognized as formateur, it will randomize between majoritarian governments ac and bc, and the joint period-two payoff of parties a and b will be $\frac{1}{2}u^a(x')+\frac{1}{2}u^b(x')+\left(-\frac{3}{4}\right)$. The probability of this event is one-half. If either party a or b is recognized as formateur, a majoritarian government ab will be formed and the joint expected payoff of parties a and b

in the second period will be $-\frac{1}{2}$. The probability of this event is also one-half. Therefore, the joint discounted expected utility of parties a and b in the first period is

$$\sum_{i=a,b} U^i\left(x'\right) = \sum_{i=a,b} u_i\left(x'\right) + \beta\left[\tfrac{1}{2}\left(-\tfrac{1}{2}\right) + \tfrac{1}{2}\left(\frac{1}{2}u^a\left(x'\right) + \frac{1}{2}u^b\left(x'\right) + \left(-\tfrac{3}{4}\right)\right)\right],$$

and

$$\begin{aligned}\sup_{x'\in R_8^T}\sum_{i=a,b} U^i\left(x'\right) &= \sup_{F_{ab}(h,w)\in R_8^T} -\tfrac{3(4+\beta)}{8}h^2 - \tfrac{4+\beta}{2}w^2 - \tfrac{\beta}{2}\left|w\right| - \tfrac{2+3\beta}{4} \\ &= -\tfrac{16+19\beta}{24} < \sup_{x'\in R_2^T}\sum_{i=a,b} U^i\left(x'\right),\end{aligned}$$

for all m and β. Therefore, $H_a\left(ab\right) \notin R_8^T$.

Region R_9^T Suppose that government ab chooses $\overline{z}$. Then in the second period every party i will be recognized with probability one-third and as formateur will randomize between majoritarian governments ij and ik, $j \neq k$. This implies that the joint discounted expected utility of parties a and b in the first period is

$$\begin{aligned}&\sum_{i=a,b} u^i\left(\overline{z}\right) + \tfrac{1}{3}\beta\left(u^a\left(\overline{z}\right) + \left(-\tfrac{3}{4}\right)\right) + \tfrac{2}{3}\beta\left(\tfrac{1}{2}\left(-\tfrac{1}{2}\right) + \tfrac{1}{2}\left(\left(-\tfrac{1}{2} - u^c\left(\overline{z}\right)\right) + \left(-\tfrac{3}{4}\right)\right)\right) \\ =\ & -\tfrac{4+5\beta}{6} < \sup_{x'\in R_2^T}\sum_{i=a,b} U^i\left(x'\right),\end{aligned}$$

for all m and β. Therefore, $H_a\left(ab\right) \notin R_9^T = \left\{\overline{z}\right\}$.

Region R_{10}^T Suppose that a policy $x' \in R_{10}^T$ is chosen. By Lemma 2, in the second period all parties will be represented and party c will get a majority vote share and as formateur form a consensus government with policy $\overline{z}$. This implies that the joint period-two payoff of parties a and b will be $\sum_{i=a,b} u^i\left(x'\right)$. Therefore, the joint discounted expected utility of parties a and b in the two periods is $\sum_{i=a,b} U^i\left(x'\right) = \left(1+\beta\right)\sum_{i=a,b} u^i\left(x'\right)$, and

$$\max_{x'\in R_{10}^T}\sum_{i=a,b} U^i\left(x'\right) = -\left(1+\beta\right) < \sup_{x'\in R_2^T}\sum_{i=a,b} U^i\left(x'\right),$$

for all m and β. Therefore, $H_a\left(ab\right) \notin R_{10}^T$.

Comparison This analysis has shown that $H_a\left(ab\right) \notin R_r^T$ for $r = 3, 4, ..., 10$. The analyses of local maxima in regions R_1^T and R_2^T implies that for all $m \in \left[0, \frac{1}{4}\right)$,

$$H_a\left(ab\right) = \begin{cases} F_{ab}\left(\frac{(1-m)\beta}{2-(1-2m)\beta}, 0\right) & \text{if } \beta \in \left[0, \beta^*\left(m\right)\right) \\ F_{ab}\left(\kappa\left(\frac{\beta}{2-\beta}\right), \left|\kappa - 1\right|\left(\frac{1}{2}\right)\right) & \text{if } \beta \in \left[\beta^*\left(m\right), 1\right). \end{cases}$$

Characterization of $\beta^*(m)$ Consider the claim that $\sup_{x' \in R_2^T} \sum_{i=a,b} U^i(x') \leq \max_{x' \in R_1^T} \sum_{i=a,b} U^i(x')$ for all $\beta \in [\beta^*(m), 1)$ and all $m \in [0, \frac{1}{4})$, where $\beta^*(m)$ is a decreasing function in m and $\beta^*(m) \in \left(0, \widehat{\beta}(m)\right)$ for all m. To show this, first of all, for all m and all $\beta \in \left[\widehat{\beta}(m), 1\right)$,

$$\begin{aligned} & \sum_{i=a,b} U^i\left(H_a\left(ab|R_1^T\right)\right) - \sum_{i=a,b} U^i\left(H_a\left(ab|R_2^T\right)\right) \\ = & \left(\left(1 + \tfrac{\sqrt{3}}{2}\right) m - \tfrac{\sqrt{3}}{2} + \tfrac{3}{4}\right)\beta + \sqrt{2\left(\left(\beta - \tfrac{1}{2}\right)^2 + \tfrac{3}{4}\right)} - 1 > 0. \end{aligned}$$

Second, for all $m \in [0, \frac{1}{4})$ and all $\beta \in \left[0, \widehat{\beta}(m)\right)$,

$$\begin{aligned} & \textstyle\sum_{i=a,b} U^i\left(H_a\left(ab|R_1^T\right)\right) \geq \sum_{i=a,b} U^i\left(H_a\left(ab|R_2^T\right)\right) \\ \Leftrightarrow \quad & 4\left(2 - (1-2m)\beta\right)\left(2\left(\beta - \tfrac{1}{2}\right)^2 + \tfrac{3}{2}\right)^{\frac{1}{2}} \geq \left(7 - 12m - m^2\right)\beta^2 - 2\left(7 - 4m\right)\beta + 12, \end{aligned}$$

which is equivalent to which is equivalent to

$$\begin{aligned} \Omega(\beta) \equiv & -\left(17 - 40m + 2m^2 + 24m^3 + m^4\right)\beta^4 + 4\left(9 - 16m + 9m^2 + 4m^3\right)\beta^3 \\ & -4\left(19 - 32m - 22m^2\right)\beta^2 + 16\left(5 + 4m\right)\beta - 16 \\ \geq & \ 0. \end{aligned}$$

Note that (1) $\Omega(0) < 0$, (2) $\lim_{\beta \to \infty} \Omega(\beta) < 0$, (3) $\Omega(1) = 7 + 168m + 122m^2 - 8m^3 - m^4$ and (4) $\Omega(\beta) = 0$ is a biquadratic equation with four roots. There are standard procedures of solving a biquadratic equation, and it can be verified that (5) two of its roots are real and the other two are imaginary. Call the two real roots β_1^* and β_2^* such that $\beta_1^* \leq \beta_2^*$. By (1), (2), (3) and (5), it follows that $0 < \beta_1^* < 1 < \beta_2^*$, and for all $\beta \in [\beta_1^*, 1)$, $\Omega(\beta) \geq 0$ and therefore $\sum_{i=a,b} U^i\left(H_a\left(ab|R_1^T\right)\right) - \sum_{i=a,b} U^i\left(H_a\left(ab|R_2^T\right)\right)$. Then, define $\beta^*(m) = \beta_1^*$ for all m. By (A), $\beta^*(m) \in \left(0, \widehat{\beta}(m)\right)$ for all m. Finally, it can be verified that the relationship between $\beta^*(m)$ and m is as shown in Figure 8. ■

Proof of Proposition 4. Consider a single-party government formed by party c in the first period. Party c chooses a policy to maximize its expected discounted sum of utilities. To analyze this maximization problem, partition the policy space into two regions: $R_1^S \equiv \overline{D}^a \setminus \left(D^b \cup D^c\right)$, $R_2^S \equiv \mathfrak{R}^2 \setminus R_1^S$.

Suppose that party c is restricted to choose a policy x' from the set of R_1^S. Then in the second period the parliamentary election leads to a majority parliament, and the majority

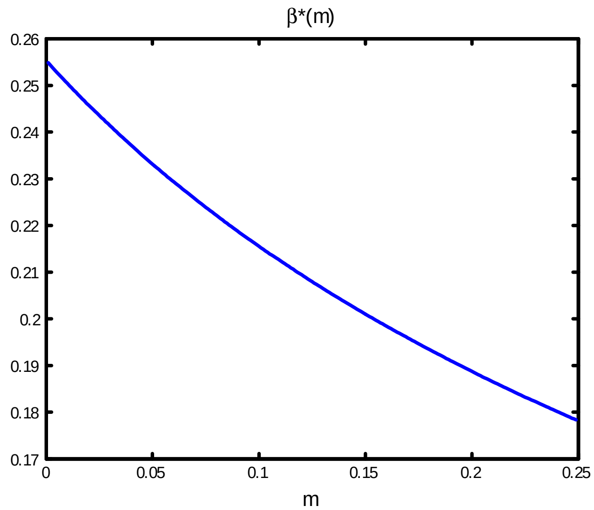

Figure 8: $\beta^*(m)$

party c forms a consensus government with policy $\overline{z}$. This implies that party c's expected discounted sum of utility is

$$\begin{aligned} U^c(x') &= u^c(x') + \beta\left[(-1) - u^b(x') - u^c(x')\right] \\ &= \tfrac{3}{4}(2\beta - 1)h^2 - \tfrac{3}{2}h + (2\beta - 1)w^2 - \tfrac{3}{4} - \tfrac{1}{2}\beta, \end{aligned}$$

where $h, w \in \Re$ are such that $F(h, w) = x'$. The first-order condition for h is

$$\frac{\partial U^c}{\partial h} = \tfrac{3}{2}\left[(2\beta - 1)h^2 - 1\right] \leq 0.$$

For $\beta \geq \frac{1}{2}$, the policy is as extreme as possible while still leading to a consensus government in the second period only for c. That is, $h^* = \widehat{h} \equiv -1 - \sqrt{\frac{2}{3}}$. For $\beta < \frac{1}{2}$, the maximum is attained at an interior solution $h^* = -\frac{1}{1-2\beta}$ if $\beta \leq \beta^o \equiv \frac{1}{2}\left(\frac{\sqrt{2}}{\sqrt{3}+\sqrt{2}}\right)$, and at a corner solution $h^* = \widehat{h}$ otherwise.

Suppose that party c is restricted to choose a policy x' from the set of R_2^S. Then compared to the case with a policy choice in R_1^S, the probability that party a is recognized as period-two formateur is no longer one. Therefore, party a loses some of its expected utility in the second period. At the same time, by choosing a policy outside R_1^S, party a makes the policy farther away from its ideal point and thus lowers its period-one utility. Thus, in equilibrium party a does not choose a policy in region R_2^S. ■

B The Measure of Social Welfare

The following proposition facilitates calculating (or approximating) the average two-period utility of all voters, given any sequence of policies (x_1, x_2).

Proposition 5 *Suppose that the ideal points of all voters are located symmetrically with respect to those of the three parties; i.e., for any voter* v_1*, whose ideal point is* $F_{ab}(h,w)$*, there exists voters* v_2 *and* v_3 *such that their ideal points are* $z^{v_2} = F_{bc}(h,w)$ *and* $z^{v_3} = F_{ca}(h,w)$*. Then, for any policy* $x \in \mathfrak{R}^2$,

$$\begin{aligned} \tfrac{1}{N}\sum_v u^v(x) &= \tfrac{1}{N}\sum_v \|z^v - \bar{z}\|^2 - \|x - \bar{z}\|^2 \\ &= \mathit{constant} + u^{v^*}(x), \end{aligned}$$

where v^* *is a hypothetical voter whose ideal point is at the center of preferences.*

Proof. Take any voter v_1 and let $(h,w) \in \mathfrak{R}^2$ be such that $F_{ab}(h,w) = z^{v_1}$. By the assumption of symmetry, there exists v_2 and v_3 such that $z^{v_2} = F_{bc}(h,w)$ and $z^{v_3} = F_{ca}(h,w)$. Define $r \equiv \|x - \bar{z}\|$, $d \equiv \|z^{v_1} - \bar{z}\| = \|z^{v_2} - \bar{z}\| = \|z^{v_3} - \bar{z}\|$, $\theta_1 = \angle x\bar{z}z^{v1}$, $\theta_2 \equiv \angle x\bar{z}z^{v2}$, and $\theta_3 \equiv \angle x\bar{z}z^{v3}$. Observe that $\theta_2 - \theta_1 = \frac{2}{3}\pi$, $\theta_1 + \theta_3 = \frac{2}{3}\pi$, and as a consequence,

$$\begin{aligned} \sum_{i=1}^{3}\cos\theta_i &= 2\cos\tfrac{1}{2}(\theta_1-\theta_2)\cos\tfrac{1}{2}(\theta_1+\theta_2) + \cos\theta_3 \\ &= 2\cos\tfrac{1}{3}\pi\cos\tfrac{1}{2}(\theta_1+\theta_2) + \cos\theta_3 \\ &= \cos\tfrac{1}{2}(\theta_1+\theta_2) + \cos\theta_3 \\ &= 2\cos\tfrac{1}{2}\left(\tfrac{1}{2}\theta_1 + \tfrac{1}{2}\theta_2 + \theta_3\right)\cos\tfrac{1}{2}\left(\tfrac{1}{2}\theta_1 + \tfrac{1}{2}\theta_2 - \theta_3\right) \\ &= 2\cos\tfrac{1}{2}\left(\tfrac{1}{2}(\theta_1-\theta_2) + (\theta_1+\theta_3)\right)\cos\tfrac{1}{2}\left(\tfrac{1}{2}\theta_1 + \tfrac{1}{2}\theta_2 - \theta_3\right) \\ &= 2\cos\tfrac{1}{2}\left(\tfrac{1}{2}\left(\tfrac{2}{3}\pi\right) + \tfrac{2}{3}\pi\right)\cos\tfrac{1}{2}\left(\tfrac{1}{2}\theta_1 + \tfrac{1}{2}\theta_2 - \theta_3\right) \\ &= 2\cos\tfrac{1}{2}\pi\cos\tfrac{1}{2}\left(\tfrac{1}{2}\theta_1 + \tfrac{1}{2}\theta_2 - \theta_3\right) \\ &= 0 \end{aligned}$$

Given policy x, voter v_i's per-period utility is

$$\begin{aligned} u^{v_i}(x) &= -\|x - z^{v_i}\|^2 \\ &= -\left[(d - r\cos\theta_i)^2 + (r\sin\theta_i)^2\right] \\ &= -\left[d^2 + r^2\left(\cos^2\theta_i + \sin^2\theta_i\right) - 2dr\cos\theta_i\right] \\ &= -\left(d^2 + r^2\right) + 2dr\cos\theta_i. \end{aligned}$$

Therefore, the aggregate per-period utility of voters v_1, v_2, and v_3 is

$$\begin{aligned}\sum_{i=1}^{3} u^{v_i}(x) &= -3\left(d^2+r^2\right)+2dr\sum_{i=1}^{3}\cos\theta_i \\ &= -3\left(d^2+r^2\right) \\ &= -\sum_{i=1}^{3}\left\|z^{v_i}-\overline{z}\right\|^2-3\left\|x-\overline{z}\right\|^2.\end{aligned}$$

■

Recall that there are sufficiently many voters, and their ideal points are uniformly distributed in a disk around the center of preferences. Therefore, voters can be grouped in triplets, such that for any voter v_1, whose ideal point is $F_{ab}(h,w)$, there exists voters v_2 and v_3 whose ideal points are *sufficiently close to* $F_{bc}(h,w)$ and $F_{ca}(h,w)$. Since the number of voters may not be a multiple of 3, there might be remainders of 1 or 2 voters. However, since there are a large number of voters, the effect of these remainder voters' utilities on the average is negligible. Therefore, $\frac{1}{N}\sum_v \left\|z^v-\overline{z}\right\|^2-\left\|x-\overline{z}\right\|^2$ is a good approximation of the average per-period utility of all voters. Note that the first term is just a constant, given any set of voters and the second term is the per-period utility of a hypothetical voter, whose ideal point is at the center of preferences. This provides a convenient measure of the social welfare.

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